# QCD thermodynamics at nonzero isospin asymmetry

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In collaboration with
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12.10.2020

# Contents

# Isospin density and chemical potential

Lattice QCD: usually set up in grand canonical ensemble

$\Rightarrow$ density $n$ $\rightarrow$ chemical potential $\mu$

- $N_f = 2 + 1$ quark chemical potentials – suitable basis for LQCD:

$$\mu_u = \mu_I + \mu_L \qquad \mu_d = -\mu_I + \mu_L \qquad \mu_s \neq 0$$

$\mu_L \neq 0 \neq \mu_s$: complex action (sign) problem

# Isospin density and chemical potential

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$$\mu_u = \mu_I \qquad \mu_d = -\mu_I \qquad \mu_s = 0$$

- pure isospin chemical potential

$\mu_L = \mu_s = 0 \;\hat{=}\;$ real action

$\longrightarrow$ suitable for importance sampling

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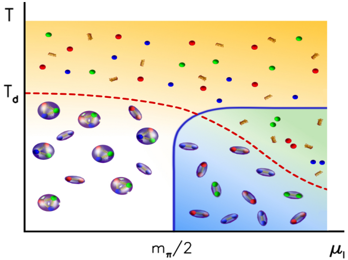
Expected phase diagram:

- hadronic phase (white)
- quark-gluon plasma




$\chi$PT [ Son, Stephanov '01 ]

[ Brandt, Endrődi, Schmalzbauer '18 ]

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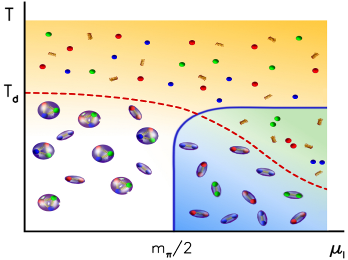
  at $T = 0$ and $\mu_I > m_\pi/2$:

  $\Rightarrow$ condensation of charged pions

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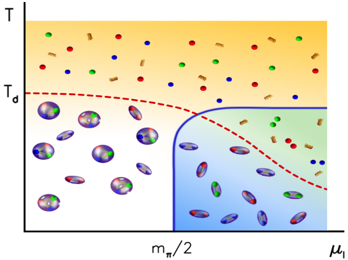
- BCS superconducting:

  pseudoscalar Cooper pairs

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# Infrared regulator – pionic source

$$\mathrm{SU}_V(2) \longrightarrow \mathrm{U}_Q(1)$$

explicit

$\mu_I \neq 0$

# Infrared regulator – pionic source

$$\mathrm{SU}_V(2) \xrightarrow[\substack{\text{explicit}\\ \mu_I \neq 0}]{} \mathrm{U}_Q(1) \xrightarrow[\substack{\text{spontaneous}\\ \mu_I \geq m_\pi/2}]{} \varnothing$$

- low mode in simulations
- cannot observe spontaneous symmetry breaking in finite $V$

# Infrared regulator – pionic source

$$SU_V(2) \xrightarrow{\text{explicit } \mu_I \neq 0} U_Q(1) \xrightarrow{\text{explicit pionic source } \lambda} \varnothing$$

- low mode in simulations
- cannot observe spontaneous symmetry breaking in finite $V$

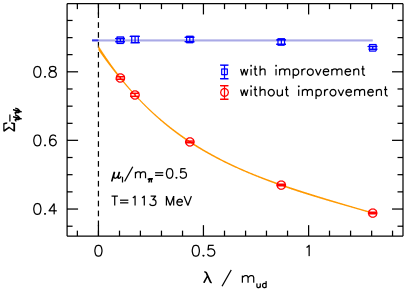
▶ need to introduce regulator: $\lambda$ – pionic source [ Kogut, Sinclair '02 ]

to obtain physical results:

extrapolate $\lambda \to 0$

reliable extrapolations:

main task for analysis



# Infrared regulator – pionic source

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explicit $\mu_I \neq 0$ — explicit pionic source $\lambda$

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to obtain physical results:

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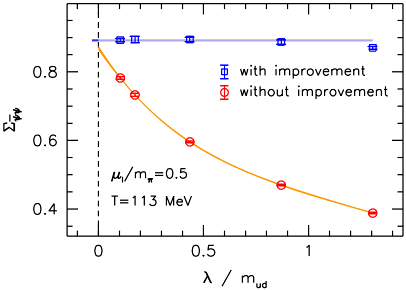
▶ improvement program:

[ Brandt, Endrődi, Schmalzbauer '18 ]

- valence quark improvement (only $u$, $d$ quark observables)
- leading order reweighting



# 2. Summary: Continuum phase diagram

# Phase diagram from the lattice

General lattice setup:

- physical $u$, $d$ and $s$ quarks
  ($N_f = 2 + 1$, convention: BEC $\mu_I = m_\pi/2$)
- use improved actions
  (2×-stout sm. stag.; Symanzik impr. gauge)

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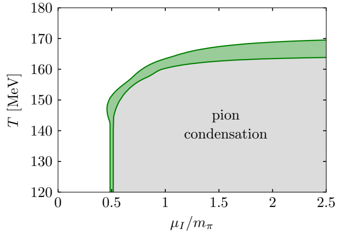
Continuum phase diagram:

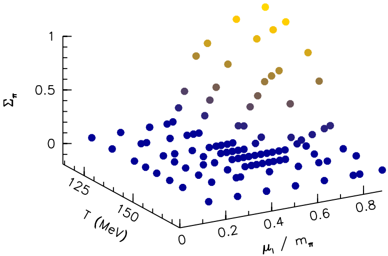
- BEC phase boundary (2nd order $O(2)$)

  order parameter:

  renormalised pion condensate

[ Brandt, Endrődi, Schmalzbauer '18 ]



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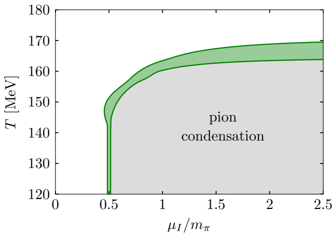
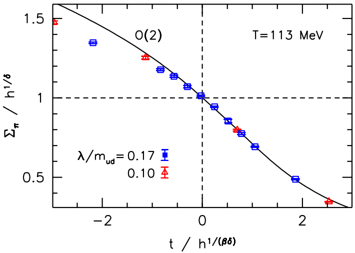
  finite size scaling:

  consistency with $O(2)$ scaling

[ Brandt, Endrődi, Schmalzbauer '18 ]



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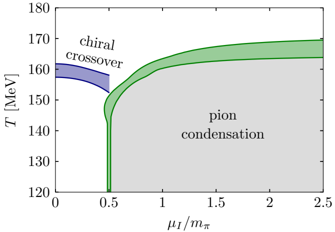
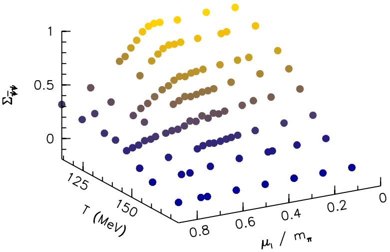
- BEC phase boundary (2nd order $O(2)$)
- chiral crossover

relevant observable:

renormalised chiral condensate

[ Brandt, Endrődi, Schmalzbauer '18 ]



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Continuum phase diagram:

- BEC phase boundary
- chiral crossover
- *pseudo-triple* point

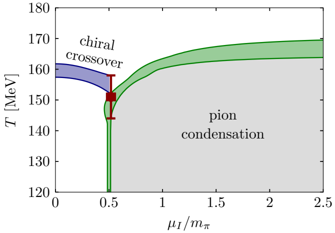
  meeting point of cross. and BEC PB:

  coexistence of three phases

  from this point on:

  phase boundaries coincide

[ Brandt, Endrődi, Schmalzbauer '18 ]



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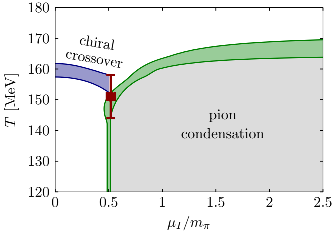


Continuum phase diagram:

- BEC phase boundary
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- *pseudo-triple* point

- BCS phase???

  work in progress . . .

# 3. Equation of State

# Equation of state at $T = 0$

Grand canonical ensemble $T = 0$:

$$p(T = 0, \mu_I = 0) = 0 \qquad n_I = \frac{\partial p}{\partial \mu_I} \qquad \epsilon = -p + n_I \mu_I$$

$$\Rightarrow \quad p(0, \mu_I) = \int_0^{\mu_I} d\mu \, n_I(0, \mu_I) \qquad I = -4p + n_I(0, \mu_I)\mu_I$$

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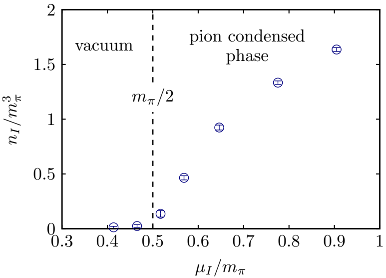
- lattice simulation: [ Brandt *et al* '18 ]

  $T = 0$ never exactly fulfilled

  $\Rightarrow$ use large $N_t$ so that $T \approx 0$



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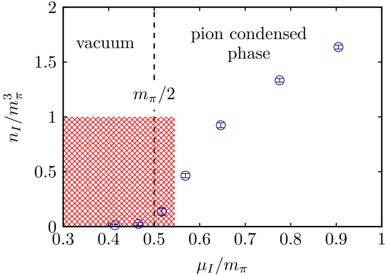
  particularly relevant for $\mu_I \approx m_\pi/2$



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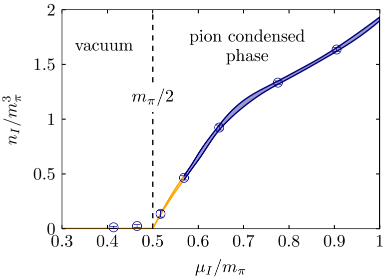
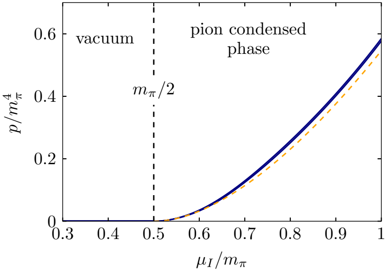
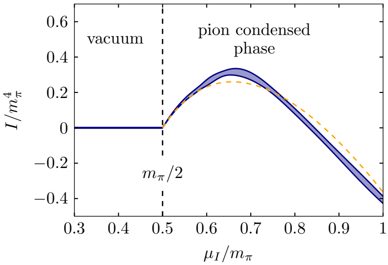
  $\Rightarrow$ fit at LO gives $f_\pi = 133(4)$



# Equation of state at $T = 0$

Resulting equation of state:




- in good agreement with NLO $\chi$PT [ Adhikari, Andersen '19 ]
- currently: only a single lattice spacing

  continuum limit: . . . work in progress

# Extracting the EoS at non-zero $T$

$p(T,0)$ and $I(T,0)$ are known [ Borsanyi *et al* '13, Bazavov *et al* '14 ]

$\Rightarrow$ need to determine:

$$\Delta p(T,\mu_I) \equiv p(T,\mu_I) - p(T,0) \quad \text{and} \quad \Delta I(T,\mu_I) \equiv I(T,\mu_I) - I(T,0)$$

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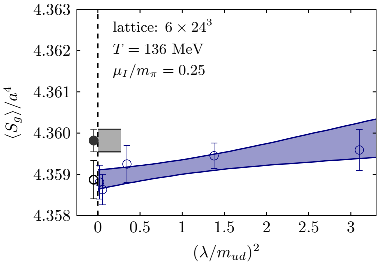

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  (ii) via a 2d interpolation of $n_I(T,\mu_I)$

  more accurate method

# 2d interpolation of $n_I(T,\mu_I)$

Starting point: $$\frac{\Delta I(T,\mu_I)}{T^4} = T\frac{\partial}{\partial T}\left(\frac{\Delta p(T,\mu_I)}{T^4}\right) + \frac{\mu_I n_I(T,\mu_I)}{T^4}$$

$$\Rightarrow \quad \Delta I(T,\mu_I) = -4\int_0^{\mu_I} d\mu_I' n_I(T,\mu_I') + \int_0^{\mu_I} d\mu_I' T\frac{\partial}{\partial T} n_I(T,\mu_I') + \mu_I n_I(T,\mu_I)$$

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terms can be computed from model independent 2d interpolation of $n_I$

model independent: all possible "good" spline fits via Monte-Carlo

[ S. Borsanyi, private comm.; Brandt, Endrődi '16 ]

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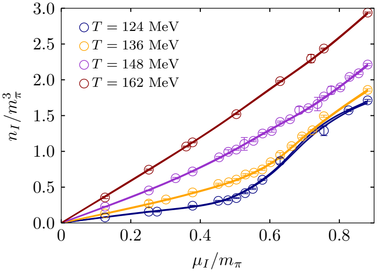
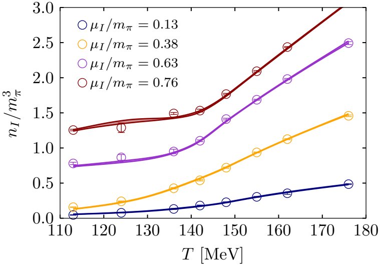
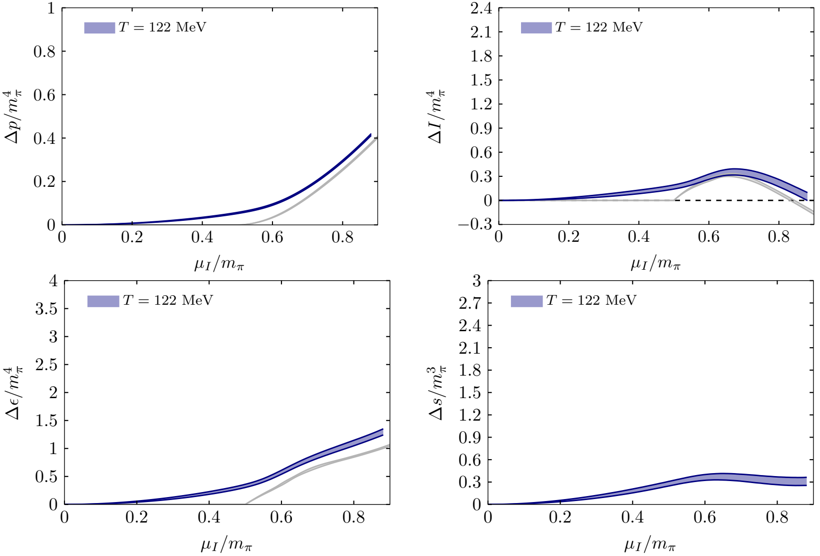


# EoS $T \neq 0$ and $\mu_I \neq 0$: $N_t = 8$



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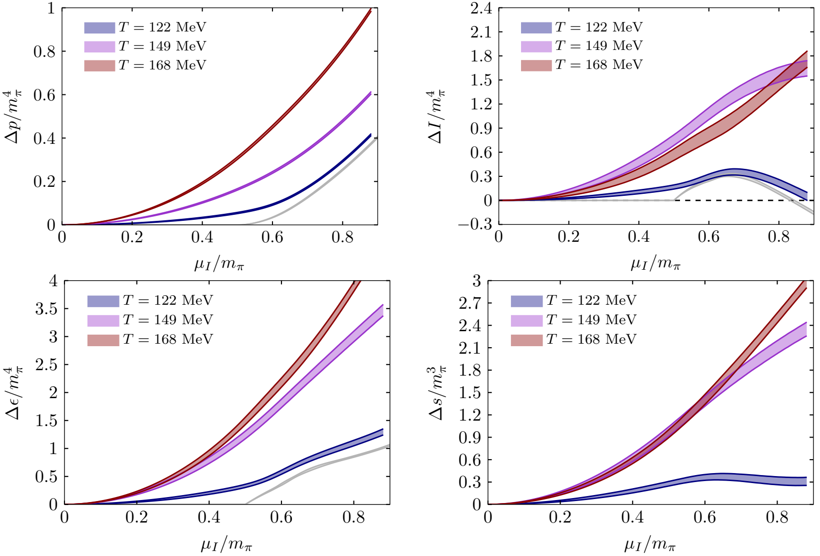
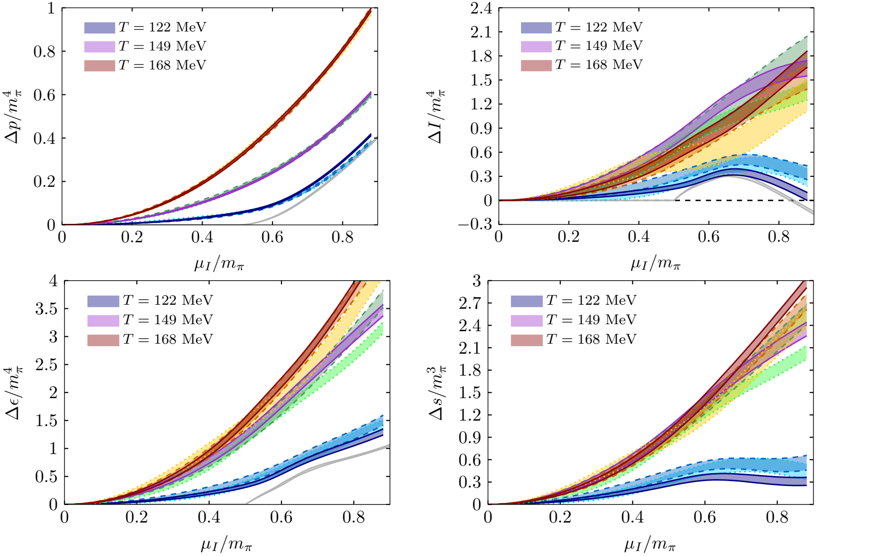


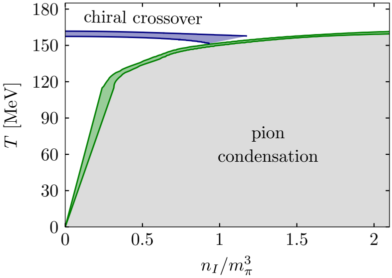
# EoS $T \neq 0$ and $\mu_I \neq 0$: cont. limit ... work in progress



# Revisiting the phase diagram: $n_I - T$ plane

using the 2d spline interpolation:

determine the phase diagram in terms of $n_I$



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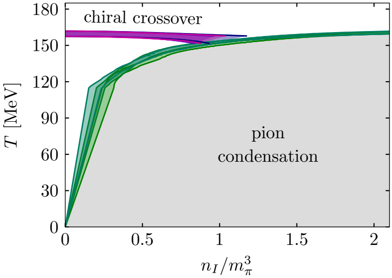


- consider multiple $N_t$ for continuum limit

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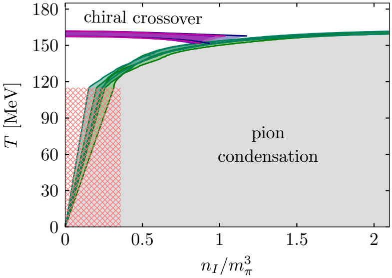


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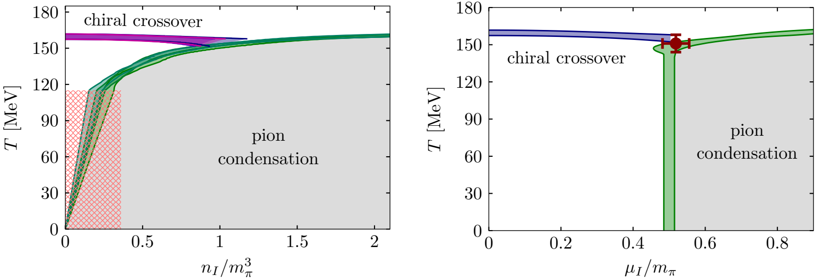


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# 4. Pion condensation in the early universe

in collaboration with:

Volodymyr Vovchenko, Fazlollah Hajkarim

and Jürgen Schaffner-Bielich

arXiv:2009.02309

# Evolution of the early universe

- isentropic expansion with parameters
  $T, \;\; \mu_B, \;\; \mu_Q, \;\; \mu_{L_\ell}$ with $\ell \in (e, \mu, \tau)$
- conservation equations:
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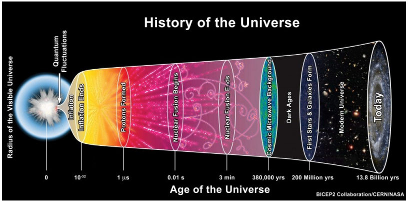


[ Credit: BICEP2 collaboration/CERN/NASA* ]

* taken from the Keck website: `https://www.keckobservatory.org`

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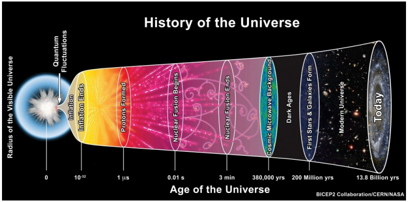

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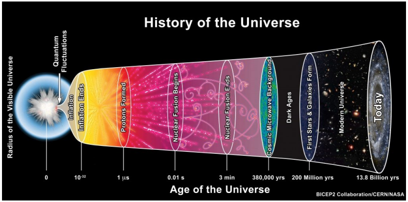


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Lattice QCD (+ Taylor)

ideal gas $\ell$ and $\nu_\ell$

ideal gas

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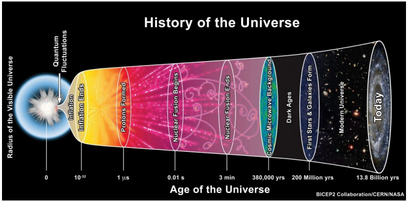


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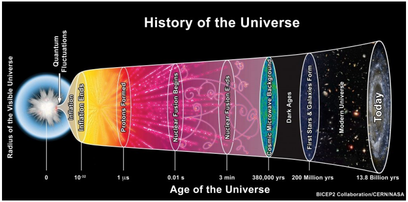
  - neutrino oscillations only for $T \lesssim 10$ MeV

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# Relevant parameters and effective mass model

[ arXiv:2009.02309 ]

Physical basis for QCD:

$$\mu_u = \frac{2}{3}\mu_Q + \frac{1}{3}\mu_B \qquad \mu_d = -\frac{1}{3}\mu_Q + \frac{1}{3}\mu_B \qquad \mu_s = -\frac{1}{3}\mu_Q + \frac{1}{3}\mu_B + \mu_S$$

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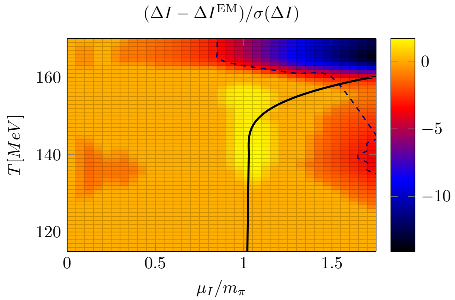
For a first look: construct a simple model which captures pion condensation

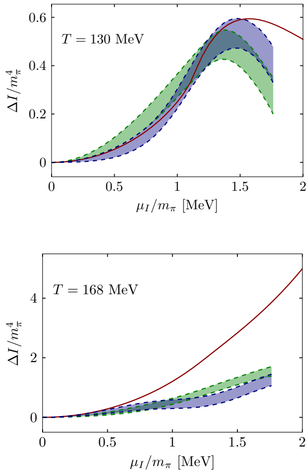
- here: use HRG combined with an effective mass model for pions (quasiparticle picture with rearrangement term)
- match the model to lattice QCD at $T = 0$

# EoS at pure $\mu_I$: model vs. lattice

[ arXiv:2009.02309 ]

Range of applicability: compare model results to lattice data ($N_t = 10, 12$)




- expect the model to work reliably for:

  $T \lesssim 160$ MeV $\quad \mu_I \lesssim 1.5\, m_\pi$

# Trajectories with lepton flavour asymmetries

[ arXiv:2009.02309 ]

lepton flavour asymmetry: $l_e + l_\mu + l_\tau = 0$ but individual $l_\ell$ not (fix $b$)

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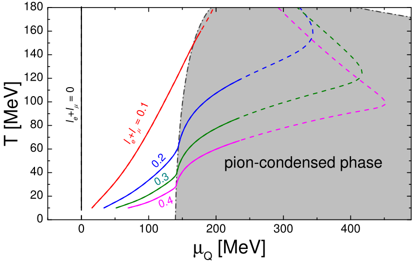

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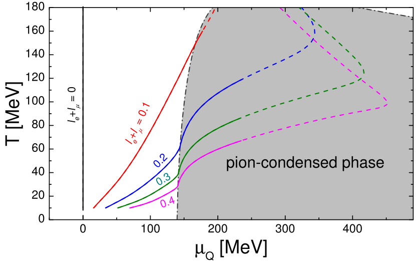
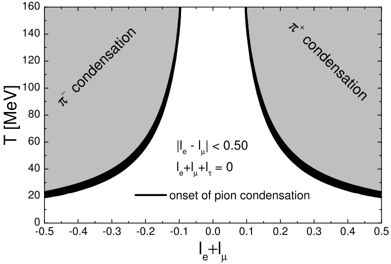


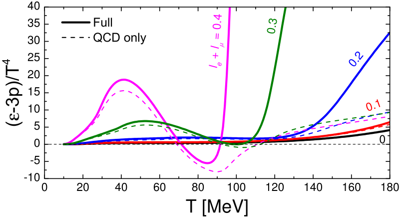

- results do not depend much on $l_e - l_\mu$ $\Rightarrow$ typically set $l_e = l_\mu$
- outside of BEC phase:

  reproduce results from [ Middeldorf-Wygas, Oldengott, Bödeker, Schwarz '20 ]

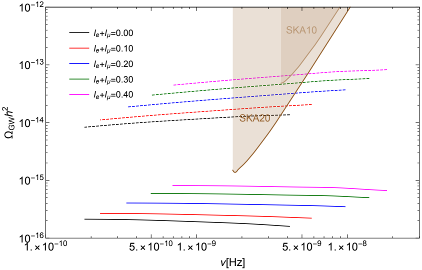
  (different model – pion condensation physics not included)

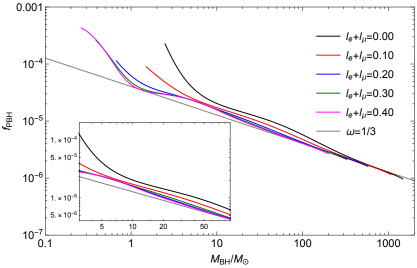
# Effects of pion condensation

[ arXiv:2009.02309 ]




- pion condensation strongly affects EoS
- enhances relic density of primordial gravitational waves
- modifies fraction of primordial black holes heavier than solar masses



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