Magnetized QCD phase diagram from the point of view of chiral symmetry restoration.

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Outline.

- Motivation.
- Magnetic fields.
- Linear Sigma Model with quarks.
- QCD phase diagram and finite eB.
- Results.

A grand opportunity Era.

- By colliding heavy ion nuclei, RHIC and LHC are making little "Big Bang matter" (the universe microseconds after the Big Bang).
- Using current detectors (PHENIX, STAR, ALICE, ATLAS and CMS) scientist are answering questions about the microsecond-old universe that cannot be addressed by any conceivable astronomial observation. And
- The properties of the matter that filled the early universe turn out to be INTERESTING (Quark-Gluon Plasma).
- What's next? Hadronic matter under extreme conditions, not only high temperatures, also high densities. We are waiting for **NICA**, FAIR, J-PARC!!!

What does QCD describe?

It is an experimental fact that in the world around us, quarks and gluons occur only in colorless, heavy package:

- Protons, neutron, ... (Baryons).
- Pions, Kaons, ... (Mesons).

They, in turn, make up everything from nuclei to neutron stars, and thus most of the mass of you and me.

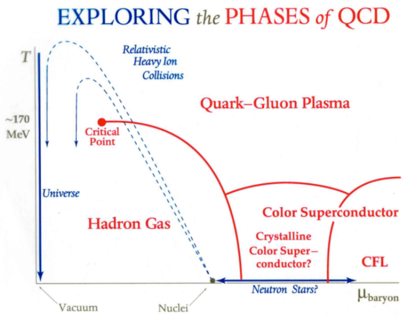
Why study QCD? Why is it a challenge?

- The only example we know of strongly interacting gauge theory.
- We "understand" the theory at short distances.
- The quasiparticles (the excitations of the vacuum) are hadrons, which do not look at all like the short distance quark and gluon degrees of freedom.

How do we respond to the challenge?

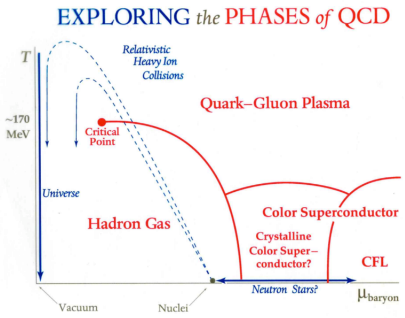
- Study the spectrum, properties and structure of the hadrons.

- Get away from vacuum. Understand other phases of QCD and their quasiparticles, map the QCD phase diagram.



by K. Rajagopal

QCD phase diagram.



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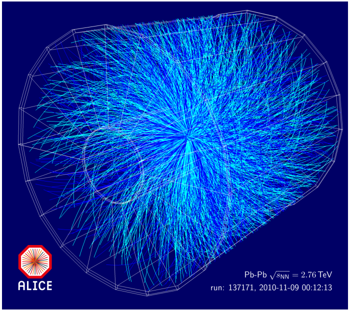
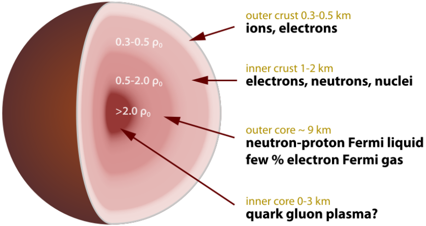
- Phase transition ⇔ Symmetry restored/broken.
  - Deconfinement and/or Chiral symmetry restoration.

Creating the Quark-Gluon Plasma.

High Temperature.



High Density.

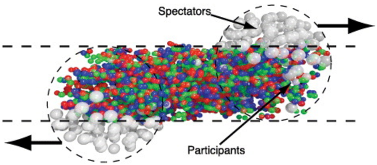


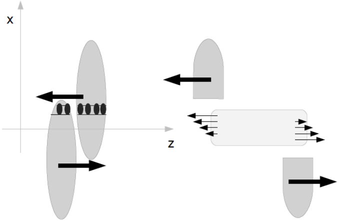
-Are these kind of systems isolated?

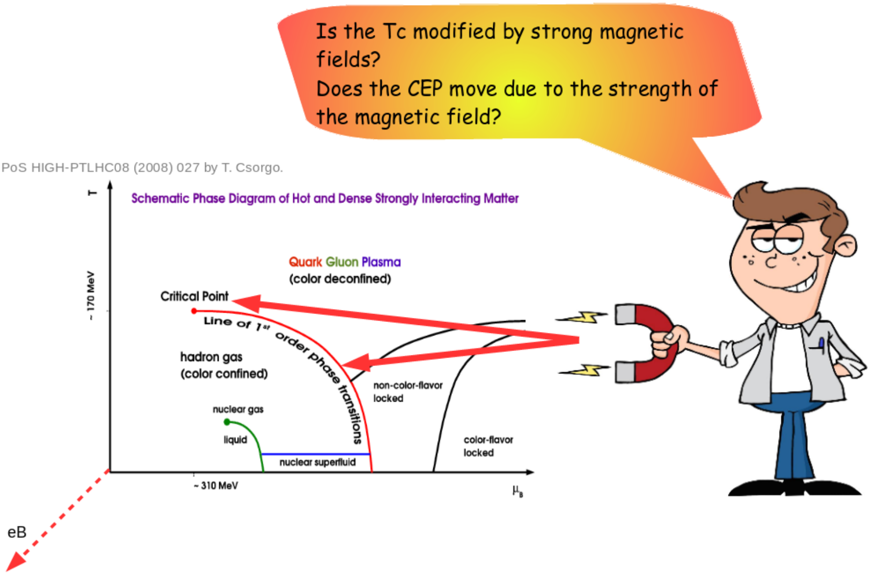
-NO.

Hadronic matter in presence of strong magnetic field.

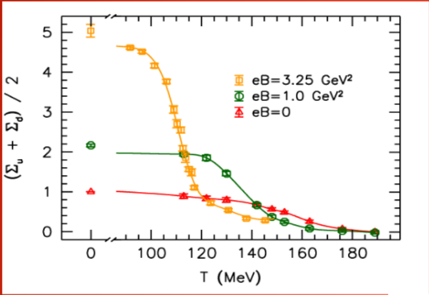
- The earth's magnetic field $\rightarrow 0.6$ Gauss.
- A common hand-held magnet $\rightarrow 100$ Gauss.
- The strongest steady magnetic fields achieved so far in the laboratory $\rightarrow 4.5 \times 10^5$ Gauss.
- Surface field of magnetars $\rightarrow 10^{15}$ Gauss.
- Heavy Ion Collisions, the strongest magnetic field ever achieved in the laboratory $\rightarrow 10^{18}$ Gauss $\approx m_\pi^2$ ($m_\pi \approx 140\ MeV$).





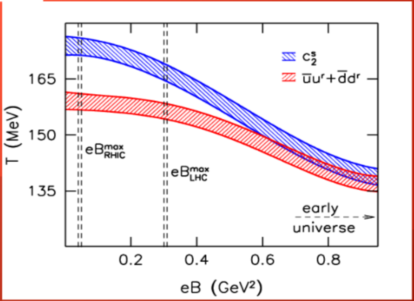


The Magnetized QCD Phase Diagram



At T=0 → Magnetic Catalysis.

Chiral Symmetry Restoration in the presence of magnetic fields → Inverse Magnetic Catalysis.



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QCD phase transition & $eB \neq 0$.

- Effective approach, using the Linear Sigma Model with quarks.
- Focusing in the chiral symmetry restoration phenomena.
- Compute the effective potential beyond the mean field approximation ($T \neq 0$, $\mu \neq 0$ and $eB \neq 0$).
- Construct the magnetized effective QCD phase diagram.

Linear Sigma Model with quarks.

- Effective model for low-energy QCD.
- Renormalizable theory.
- Implement ideas of chiral symmetry $(SU(2)_L \times SU(2)_R \to O(4))$.
- Effects of quarks and mesons on the chiral phase transition.
- Spontaneous symmetry breaking $O(4) \to O(3)$.

Linear Sigma Model with quarks.

Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(D_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu D_\mu \psi - g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi,$$

with

$$D_\mu = \partial_\mu + iqA_\mu.$$

To allow for spontaneous symmetry breaking

$$\sigma \to \sigma + v,$$

v can later identified as the order parameter of the theory.

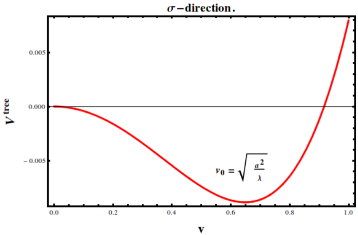
Linear Sigma Model with quarks.

After the shift

$$\mathcal{L} = -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma] - \frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2$$
$$- \frac{1}{2}[\vec{\pi}(\partial_\mu + iqA_\mu)^2\vec{\pi}] - \frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2$$
$$+ i\bar{\psi}\gamma^\mu D_\mu\psi - gv\bar{\psi}\psi + \mathcal{L}_I^b + \mathcal{L}_I^f$$
$$+ \frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4$$

with masses

$$m_\sigma^2 = 3\lambda v^2 - a^2,$$
$$m_\pi^2 = \lambda v^2 - a^2,$$
$$m_f = gv.$$



Effective potential.

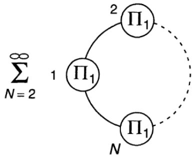
- Mean field approximation (The first quantum and thermal correction).
  - Boson and fermion fields.
  - Imaginary time formalism.

$$V_b^{(1)} = \frac{T}{2} \sum_n \int dm_b^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{ds}{\cosh(q_b B s)} \times e^{-s(\omega_n^2 + k_3^2 + k_\perp^2 \frac{\tanh(q_b B s)}{q_b B s} + m_b^2)}$$

$$V_f^{(1)} = -\sum_{r=\pm 1} T \sum_n \int dm_f^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{ds}{\cosh(q_f B s)} \times e^{-s[(\tilde{\omega}_n - i\mu)^2 + k_3^2 + k_\perp^2 \frac{\tanh(q_f B s)}{q_f B s} + m_f^2 + r q_f B]}$$

Beyond mean field (1).

Next term in the perturbative series is the ring diagrams (Dolan & Jackiw, Phys. Rev. D**12** 3320 (1974)).



Le Bellac.

Screening properties of the plasma.

$$V^{ring} = \sum_n \int \frac{d^3k}{(2\pi)^3} \ln[1 + \Pi(m_b)\Delta(\omega_n, k; m_b^2)]$$

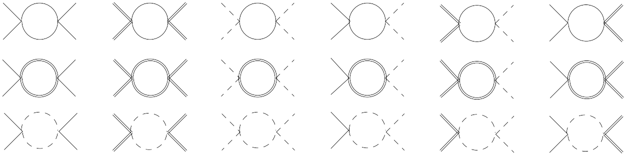
with the self-energy

$$\Pi = \lambda \frac{T^2}{2} - N_f N_c g^2 \frac{T^2}{\pi^2}[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$

Beyond mean field (2).

Effective coupling constants (Thermo-magnetic correction).

- λ coupling constant

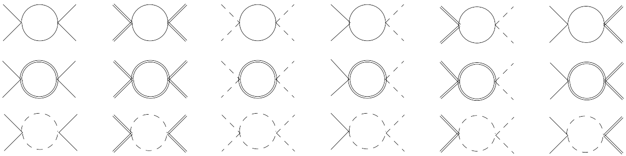


(a) (b) (c) (d) (e) (f)

Beyond mean field (2).

Effective coupling constants (Thermo-magnetic correction).

- λ coupling constant



(a) (b) (c) (d) (e) (f)

$$\lambda_{eff} = \lambda[1 + 24\lambda(9I(\Pi; m_\sigma^2) + I(\Pi; m_\pi^2) + 4J(\Pi; m_\pi^2))]$$

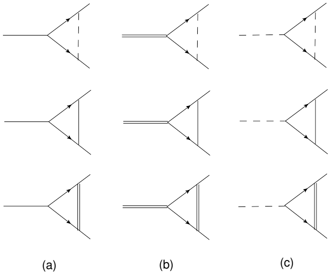
$$I(\Pi) = \frac{T}{8\pi}\frac{1}{\sqrt{\Pi}} - \frac{1}{16\pi^2}\Big[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) + 1 - 2\gamma_E + \zeta(3)\left(\frac{\sqrt{\Pi}}{2\pi T}\right)^2\Big],$$

$$J(\Pi) = \frac{T}{16\pi}\frac{1}{(2qB)^{1/2}}\zeta\left(\frac{3}{2}, \frac{1}{2} + \frac{\Pi}{2qB}\right) - \frac{1}{16\pi^2}\Big[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) + 1$$

$$- 2\gamma_E + \zeta(3)\left(\frac{\sqrt{\Pi}}{2\pi T}\right)^2\Big] - \frac{(qB)^2}{1024\pi^6 T^4}\zeta(5).$$

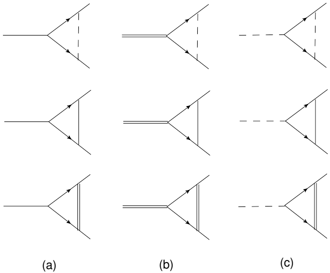
Beyond mean field (2).

- g coupling constant



Beyond mean field (2).

- g coupling constant



$$g_{eff} = g[1 + g^2(\delta\Gamma_{\mathsf{TV}} + \delta\Gamma_{\mathsf{TB}})],$$

$$\delta\Gamma_{\mathsf{TV}} = \frac{1}{8\pi^2}\left(\ln\left(\frac{a}{T\pi}\right) + \frac{\gamma_E}{2} - \frac{1}{2} - \ln(2\pi)\right),$$

$$\delta\Gamma_{\mathsf{TB}} = \frac{(q_u + q_d)B}{8\pi^2}\left(\frac{4}{3m_f^2}\right)\left(\ln(2) - \frac{\pi T}{2\sqrt{\Pi}}\right).$$

Effective potential.

$$
\begin{aligned}
V^{(eff)} = & -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 + \sum_{i=\sigma,\pi^0}\left\{\frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) - 2\gamma_E + 1\right]\right. \\
& \left. - \ \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T}{12\pi}(m_i^2 + \Pi)^{3/2}\right\} \\
& + \sum_{i=\pi_+,\pi_-}\left\{\frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) - 2\gamma_E + 1\right] - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24}\right. \\
& + \frac{T(2qB)^{3/2}}{8\pi}\zeta\left(-\frac{1}{2}, \frac{1}{2} + \frac{m_i^2 + \Pi}{2qB}\right) - \frac{(qB)^2}{192\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right)\right. \\
& \left.\left. -2\gamma_E + 1 + \zeta(3)\left(\frac{m_i}{2\pi T}\right)^2 - \frac{3}{4}\zeta(5)\left(\frac{m_i}{2\pi T}\right)^4\right]\right\} \\
& - N_c \sum_{f=u,d}\left[\frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) + \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T}\right) + \psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T}\right)\right]\right. \\
& + 8m_f^2 T^2[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})] - 32T^4[Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})] \\
& + \frac{(q_f B)^2}{24\pi^2}\left[\ln\left(\frac{(\pi T)^2}{2a^2}\right) - 2\gamma_E + 1 - \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T}\right) - \psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T}\right)\right. \\
& \left.\left. + \frac{2\pi}{((\pi + i\mu/T)^2 + m_f^2/T^2)^{1/2}} + \frac{2\pi}{((\pi - i\mu/T)^2 + m_f^2/T^2)^{1/2}} - \frac{4\pi}{(\pi^2 + m_f^2/T^2)^{1/2}}\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
V^{(eff)} = & -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 + \sum_{i=\sigma,\pi^0} \left\{ \frac{m_i^4}{64\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) - 2\gamma_E + 1 \right] \right. \\
& \left. - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T}{12\pi}(m_i^2 + \Pi)^{3/2} \right\} \\
& + \sum_{i=\pi_+,\pi_-} \left\{ \frac{m_i^4}{64\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) - 2\gamma_E + 1 \right] - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} \right. \\
& + \frac{T(2qB)^{3/2}}{8\pi} \zeta\left(-\frac{1}{2}, \frac{1}{2} + \frac{m_i^2 + \Pi}{2qB} \right) - \frac{(qB)^2}{192\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) \right. \\
& \left. \left. -2\gamma_E + 1 + \zeta(3)\left(\frac{m_i}{2\pi T} \right)^2 - \frac{3}{4}\zeta(5)\left(\frac{m_i}{2\pi T} \right)^4 \right] \right\} \\
& - N_c \sum_{f=u,d} \left[\frac{m_f^4}{16\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) + \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T} \right) + \psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T} \right) \right] \right. \\
& + 8 m_f^2 T^2 [Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})] - 32 T^4 [Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})] \\
& + \frac{(q_f B)^2}{24\pi^2} \left[\ln\left(\frac{(\pi T)^2}{2a^2} \right) - 2\gamma_E + 1 - \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T} \right) - \psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T} \right) \right. \\
& \left. \left. + \frac{2\pi}{((\pi + i\mu/T)^2 + m_f^2/T^2)^{1/2}} + \frac{2\pi}{((\pi - i\mu/T)^2 + m_f^2/T^2)^{1/2}} - \frac{4\pi}{(\pi^2 + m_f^2/T^2)^{1/2}} \right] \right]
\end{aligned}
$$

Pseudo-critical temperature

The criterion to find the temperature where the chiral symmetry is restored, is the following

$$\left.\frac{d^2V^{(eff)}}{dv^2}\right|_{v=0} = 0,$$

it means

$$curvature = mass^2,$$

and this is only valid when the restoration of the chiral symmetry is a second order phase transition.

Parameter space.

- Five parameters.
  - Two coupling constants λ and g, the critical temperature T_c^0 at $\mu = 0$, the parameter a and eB.
- Boson thermal masses

$$m_\sigma^2(T) = 3\lambda v^2 - a^2 + \frac{\lambda T^2}{2} + \frac{N_f N_c g^2 T^2}{6},$$
$$m_\pi^2(T) = \lambda v^2 - a^2 + \frac{\lambda T^2}{2} + \frac{N_f N_c g^2 T^2}{6}.$$

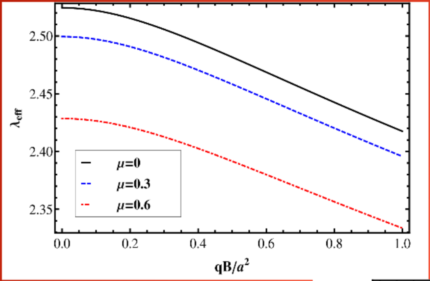
- At the phase transition, the curvature of V^{eff} vanishes for $v = 0$

$$\frac{a}{T_c^0} = \sqrt{\frac{\lambda}{2} + \frac{N_f N_c g^2}{6}}.$$

- From the vacuum boson masses, we can fix the value of a

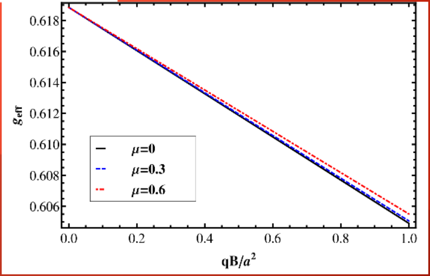
$$a = \sqrt{\frac{m_\sigma^2 - 3m_\pi^2}{2}}.$$

Results. Effective Coupling Constants.



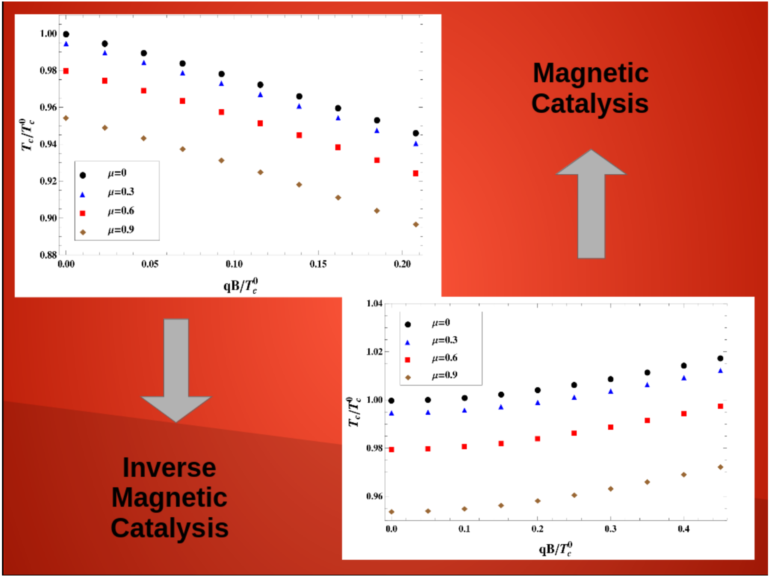
Thermo-magnetic correction

Both decrease as function of eB

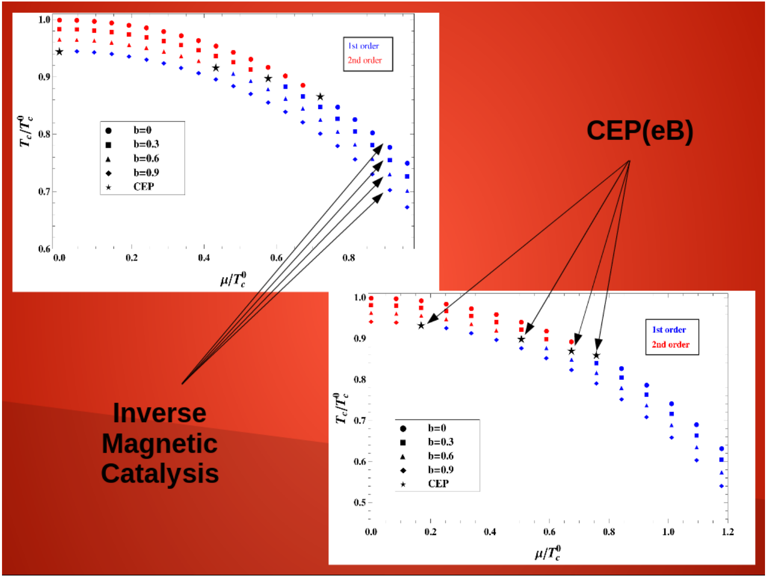


Ricardo Farias' talk (arXiv:2009.13740)

Results. Inverse Magnetic Catalysis.



Results. The tMagnetized QCD Phase Diagram.



Conclusions.

- Working in the LSMq at $eB = 0$, CEP is located in the region found by mathematical extensions of lattice analyses.
- We computed the effective potential for weak eB and included plasma screening effects through
  - The boson's self energy.
  - Thermo-magnetic corrections of the couplings.
- Corrections in the couplings are crucial to obtain inverse magnetic catalysis.
- Model shows in quantitative terms that CEP moves toward lower values of critical quark chemical potential and larger values of the critical temperature as the field intensity increases.

Interpretation.

Features of the phase diagram can be understood in general terms:

- Magnetic field produces a dimension reduction $\rightarrow$ virtual charged particles are effectively constrained to occupy Landau levels $\rightarrow$ their motion is restricted to planes $\rightarrow$ on average these particles lie closer to each other.
- Translating to QCD, quarks in the presence of eB get closer to each other $\rightarrow$ the strength of the interaction gets reduced. This phenomenon should manifest in the weakening of the quark condensate with increasing eB.

References

1. A. Ayala, C. A. Dominguez, L. A. Hernandez, M. Loewe and R. Zamora, *Magnetized effective QCD phase diagram*, Phys. Rev. D**92** (2015) no.9, 096011.

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Many Thanks!!!