Advances on the QCD phase diagram from Ward Identities and Effective Theories

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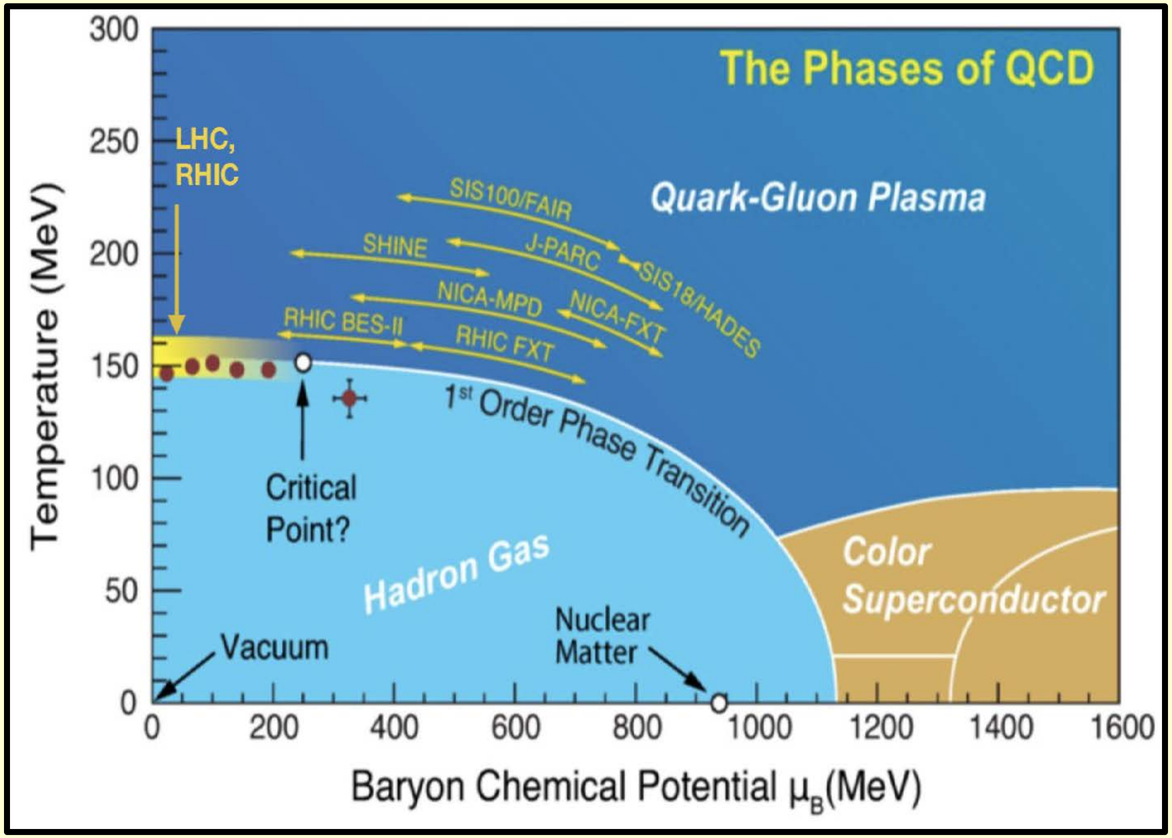
OUTLINE:

- **Aspects of the QCD phase diagram**
- **Ward Identities: chiral and $U(1)_A$ partners**
- **Effective theories: partners, susceptibilities, chiral imbalance.**

XXXII International Workshop on High Energy Physics: "Hot Problemsof Strong Interactions"

QCD PHASE DIAGRAM



A.Bazavov et al, USQCD whitepaper
EPJA 2019

QCD transition (chiral/deconfinement)

CROSSOVER transition for $\mu_B = 0$, $N_f = 2 + 1$ and physical masses

(true phase transition for $N_f = 2$ massless)

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Well established in lattice through thermodynamic observables:

- **Inflection point of (subtracted) light quark condensate** $\langle \bar{q}q \rangle_l (T)$
- **Peak of scalar susceptibility** $\chi_S(T) = -\frac{\partial}{\partial m_l} \langle \bar{q}q \rangle_l = \int_x \left[\langle \bar{q}q(x)\bar{q}q(0) \rangle_T - \langle \bar{q}q \rangle_l^2 \right]$

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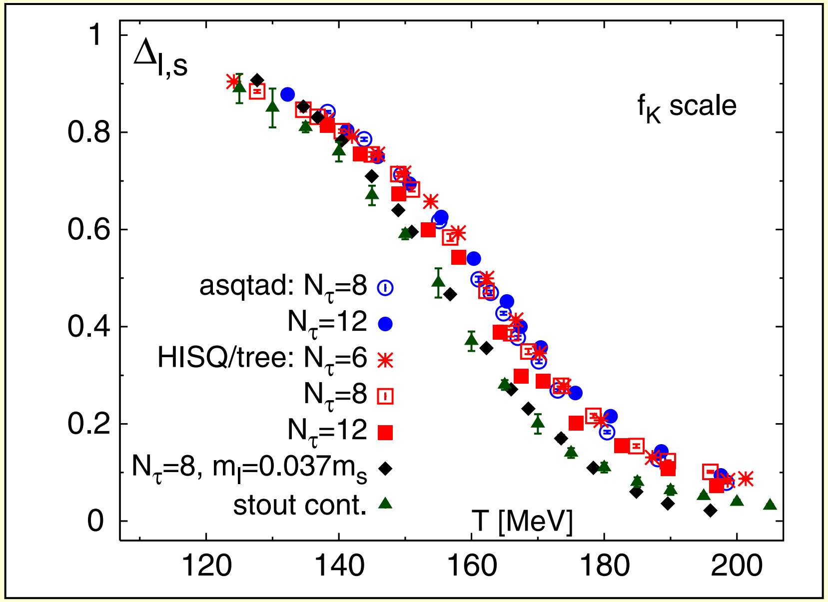
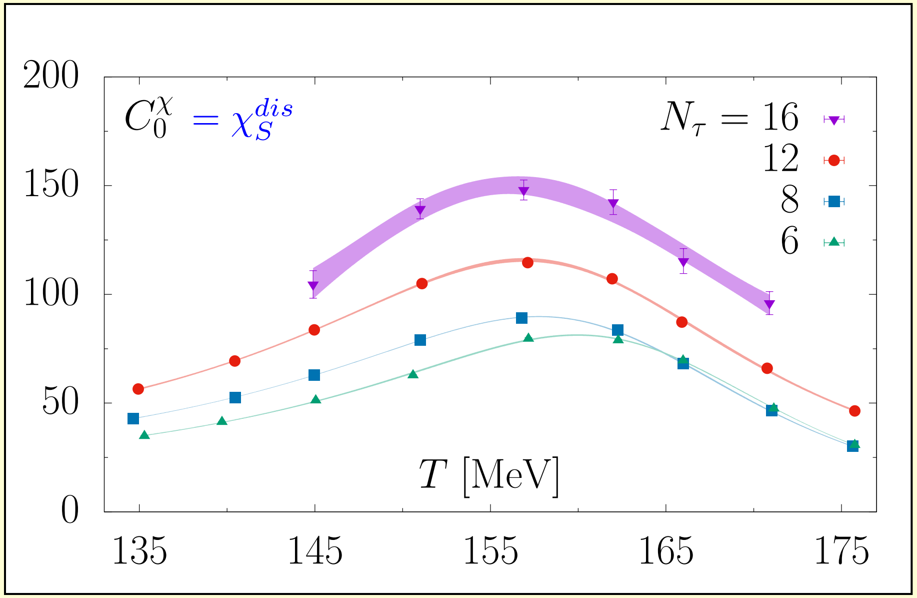
... as well as chiral partners through susceptibilities and screening masses degeneration:

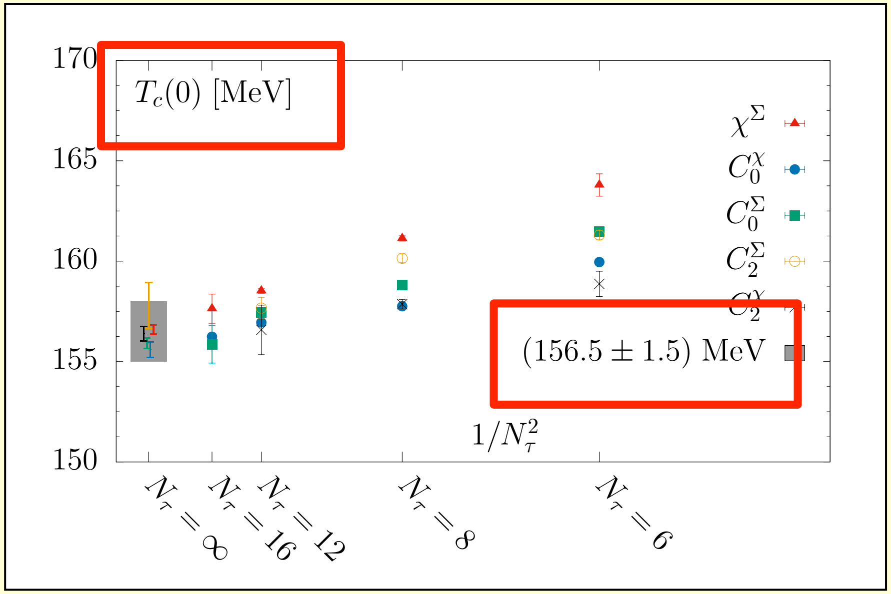
$$\rho/a_1, \ \sigma/\pi, \ \eta/a_0, \ldots$$

CONDENSATE AND SCALAR SUSCEPTIBILITY

A.Bazavov et al (Hot QCD) 2012-2019





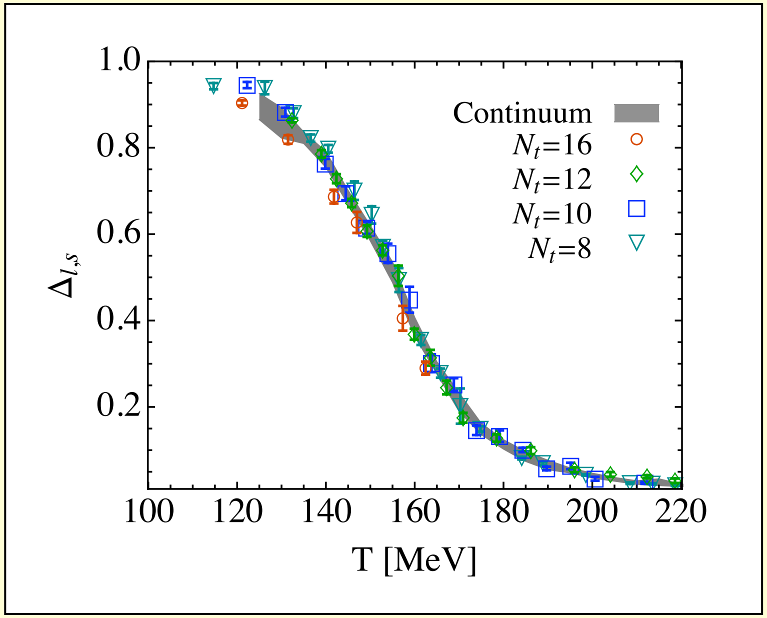


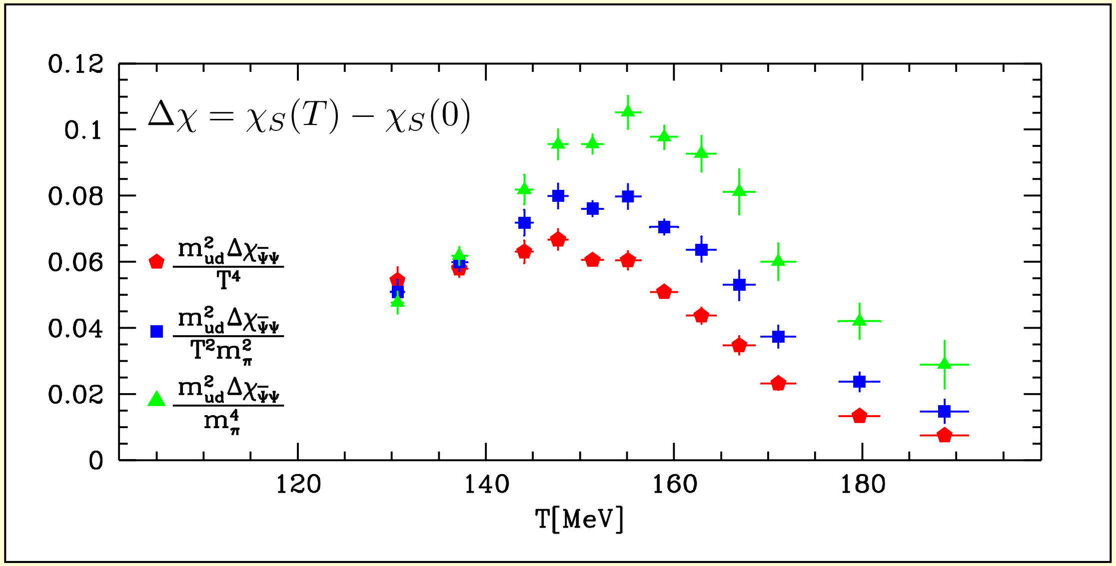
$$\Delta_{l,s} = \frac{\langle \bar{q}q \rangle_T - (2m_q/m_s)\,\langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_0 - (2m_q/m_s)\,\langle \bar{s}s \rangle_0}$$

Chiral limit $T_c^0 = 132^{+3}_{-6}$ MeV with reasonable $O(4)$ scaling (Ding et al 2019)

CONDENSATE AND SCALAR SUSCEPTIBILITY

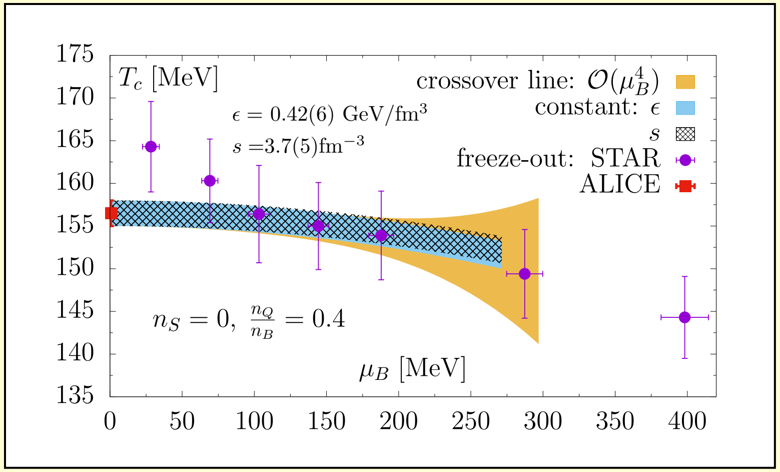
Y.Aoki, S. Borsanyi et al (Budapest-Wuppertal) 2009, 2010





$$\boxed{T_c \sim 155 \ \mathbf{MeV}}$$

QCD phase diagram explored in HIC $\rightarrow$ chemical freeze-out close to phase boundary



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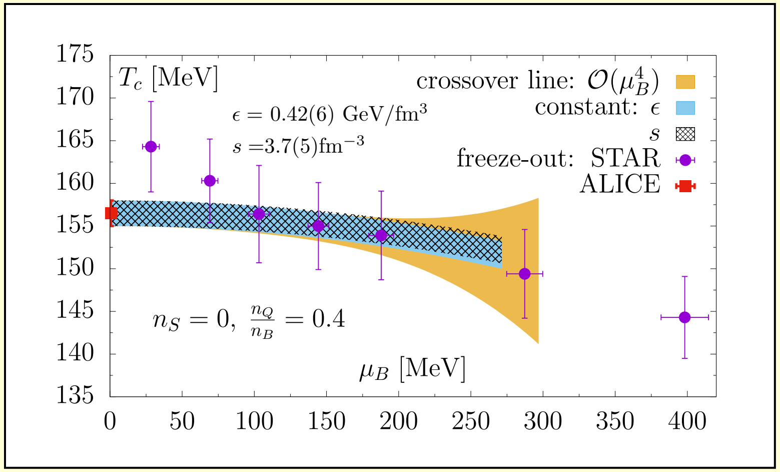

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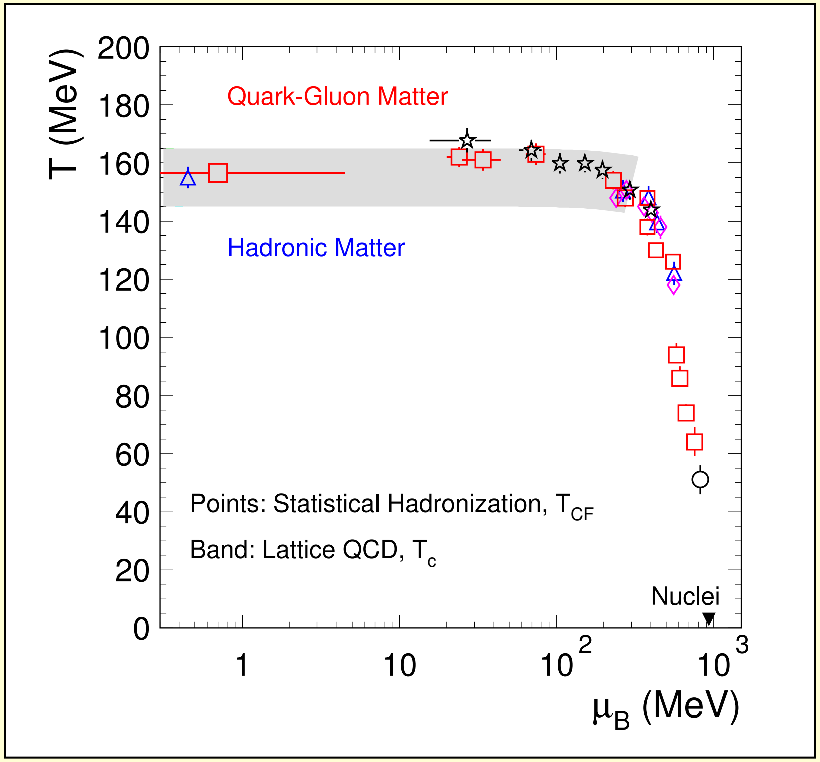
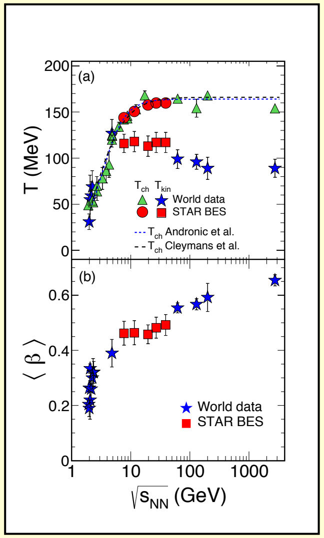
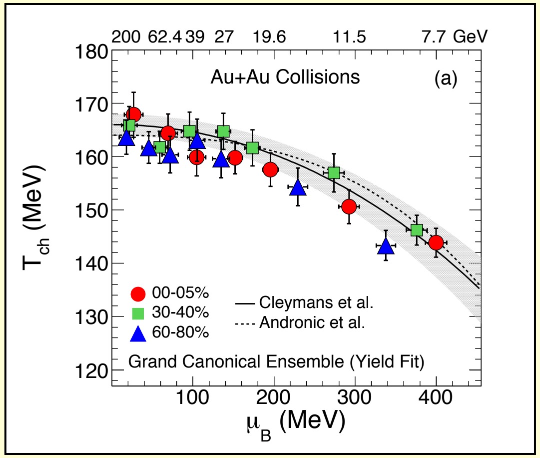


Andronic et al 2018 (ALICE)





BES (STAR Adamczyk et al 2017)

Chemical FO from Hadron Statistical Model fit to hadron yields (central ALICE data)

CHIRAL PATTERNS AND PARTNERS

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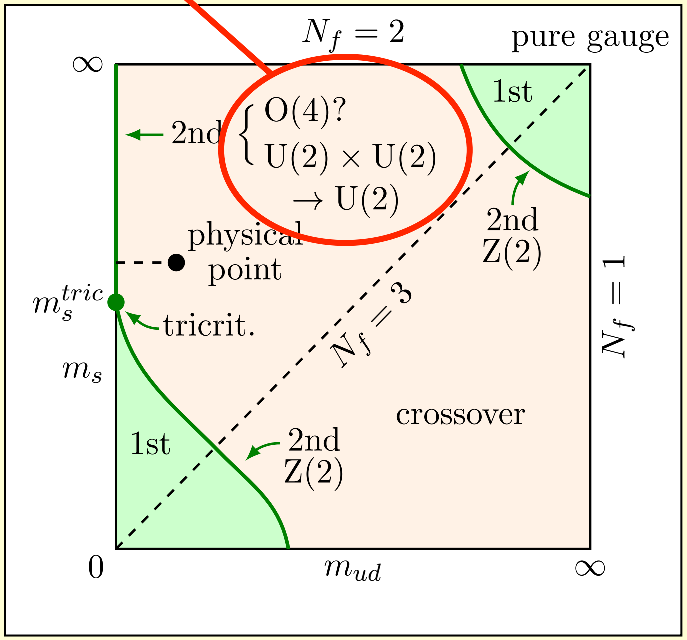
Universality class depends on the strenght of $U(1)_A$ breaking @T_c



B.B. Brandt et al 2019

Gross, Pisarski, Yaffe 1981, Shuryak 1994. Cohen 1996. Lee-Hatsuda 1996

Pisarski, Wilczek, 1984. Pelissetto, Vicari 2013. Esser, Grahl, Rischke 2015

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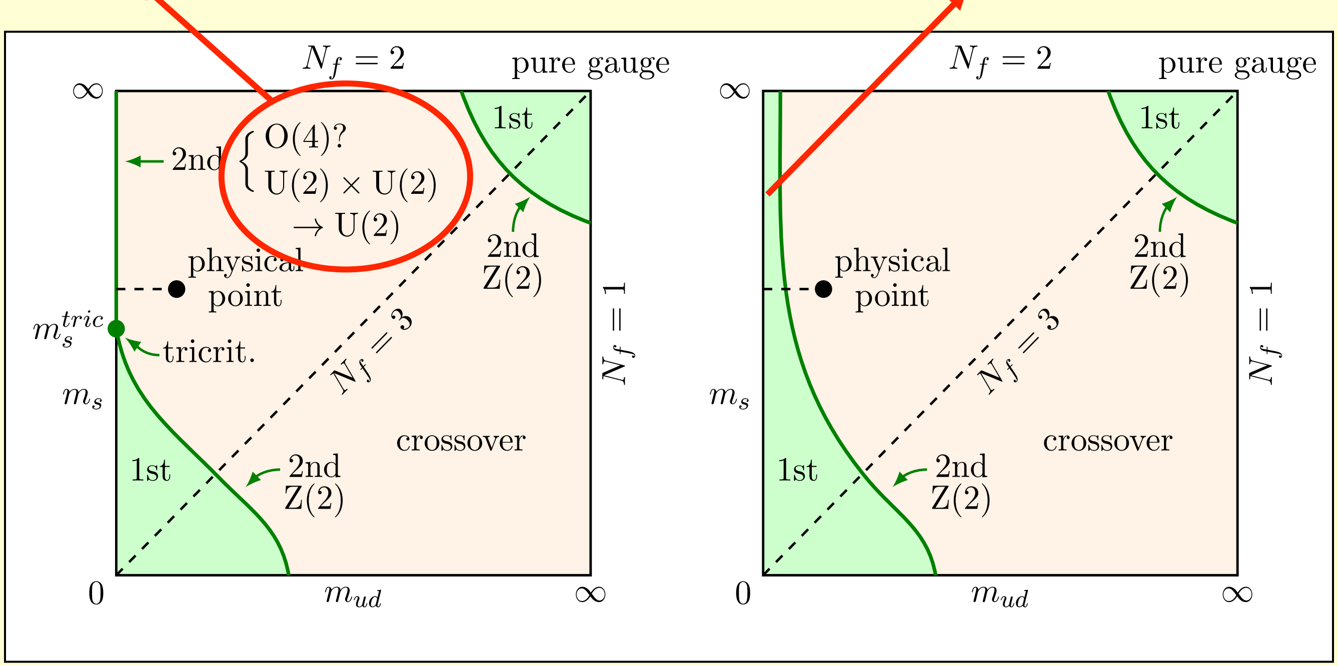

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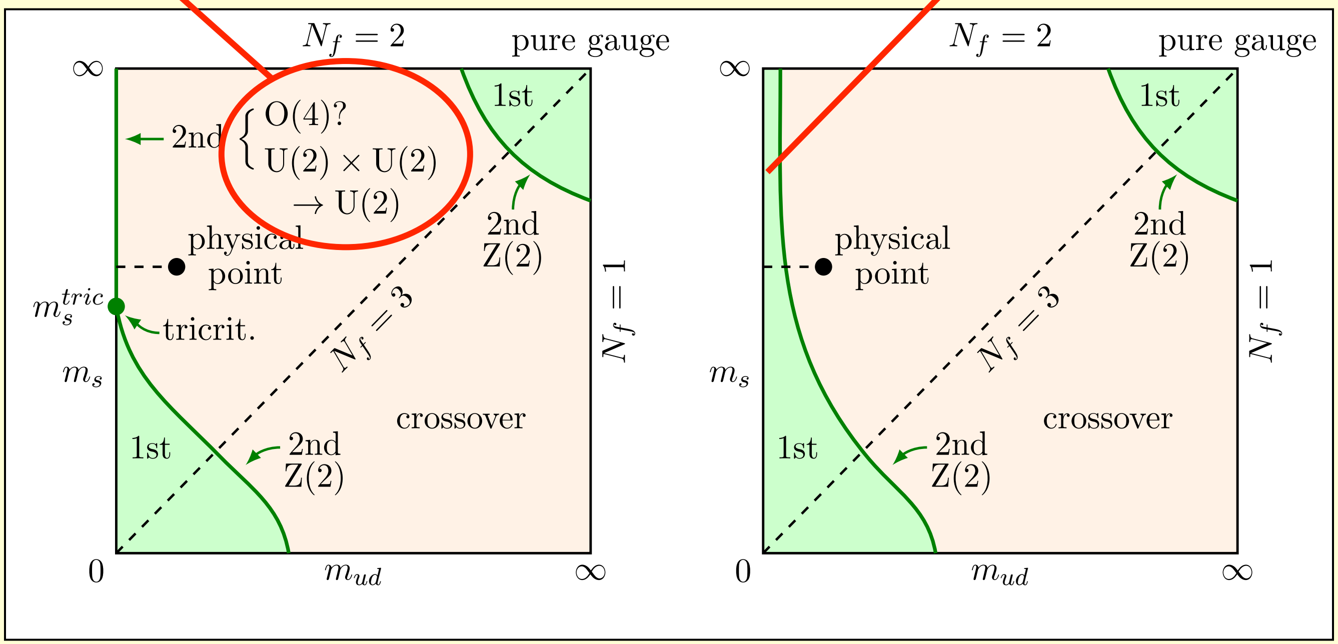


B.B. Brandt et al 2019

⇒ Is $U(1)_A$ asymptotic restoration effective already at T_c?
(specially near chiral limit)

$O(4)$ and $U(1)_A$ partners for scalar/pseudoscalar nonets: $I = 0, 1$

$\psi_l^T = (u, d)$

$I = 1$ $\qquad\qquad$ $I = 0$

$$\pi^a = i\bar{\psi}_l \gamma_5 \tau^a \psi_l \quad \overset{SU(2)_A}{\longleftrightarrow} \quad \sigma_l = \bar{\psi}_l \psi_l$$

$$\updownarrow U(1)_A \qquad\qquad\qquad \updownarrow U(1)_A$$

$$\delta^a = \bar{\psi}_l \tau^a \psi_l \quad \overset{SU(2)_A}{\longleftrightarrow} \quad \eta_l = i\bar{\psi}_l \gamma_5 \psi_l$$

Lowest meson states:

$\pi^a \to$ **pion**, $\quad \delta^a \to a_0(980)$

$\sigma_l,\ \sigma_s = \bar{s}s \to f_0(500), f_0(980)$ **(mixed)**

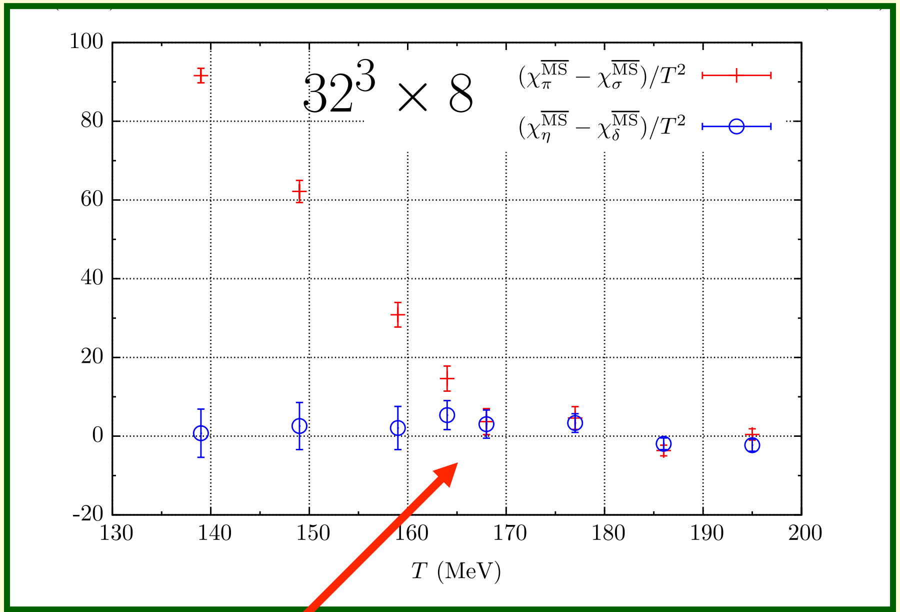
$\eta_l,\ \eta_s = i\bar{s}\gamma_5 s \to \eta, \eta'$ **(mixed)**

Chiral SSB: $SU(2)_V \times SU(2)_A\,[\approx O(4)] \to SU(2)_V\,[\approx O(3)]$

$N_f = 2 + 1$ susceptibilities
(for physical m_{ud}, m_s masses)

Buchoff et al (LLNL/RBC coll) PRD89 (2014)

$$\begin{array}{ccc} \pi^a = i\bar\psi_l\gamma_5\tau^a\psi_l & \xleftrightarrow{SU(2)_A} & \sigma_l = \bar\psi_l\psi_l \\ \updownarrow_{U(1)_A} & & \updownarrow_{U(1)_A} \\ \delta^a = \bar\psi_l\tau^a\psi_l & \xleftrightarrow{SU(2)_A} & \eta_l = i\bar\psi_l\gamma_5\psi_l \end{array}$$



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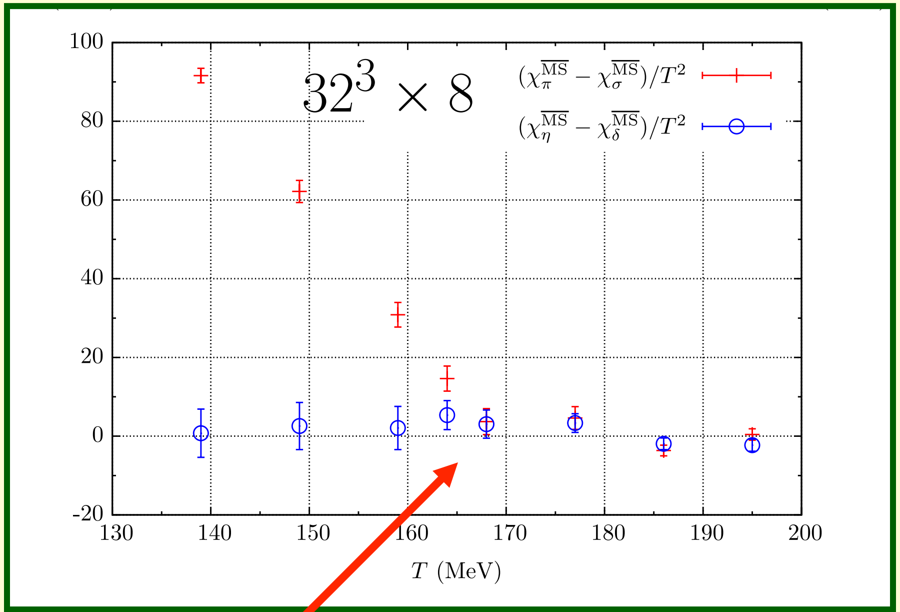

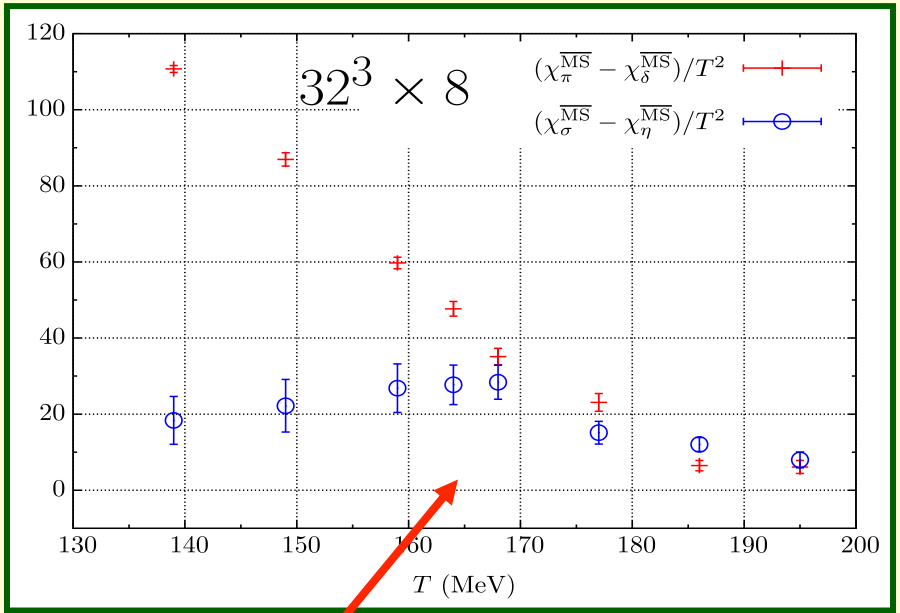

$O(4)$ **OK** (with large uncertainties in $\chi_\eta - \chi_\delta$)

significant $U(1)_A$ breaking @ T_c for physical masses

Aoki et al, 2012, Cossu et al, 2013 ($N_f = 2$, $\hat{m} \to 0$)

Brandt et al 2016, ($N_f = 2$, $\hat{m} \neq 0$)
2019 ($N_f = 2$, incl. $\hat{m} \to 0$ screening masses)

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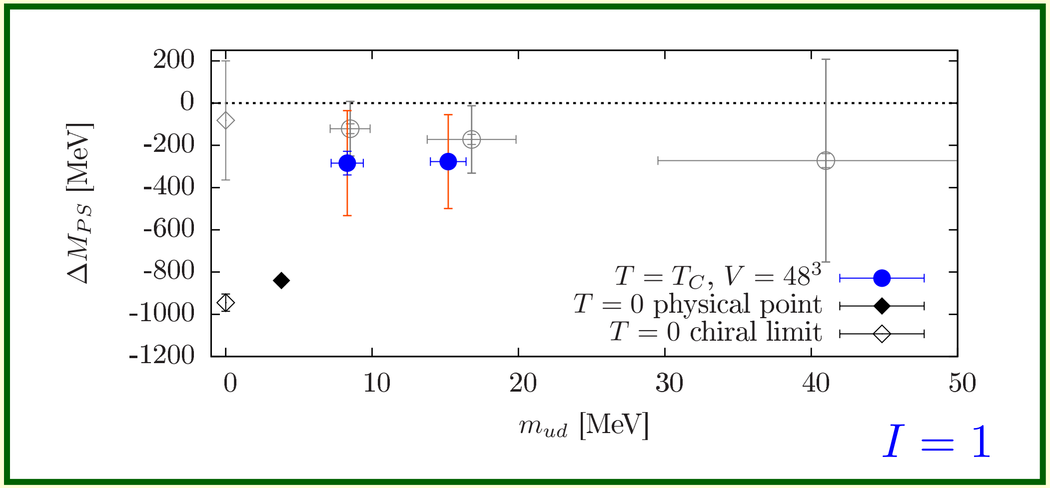

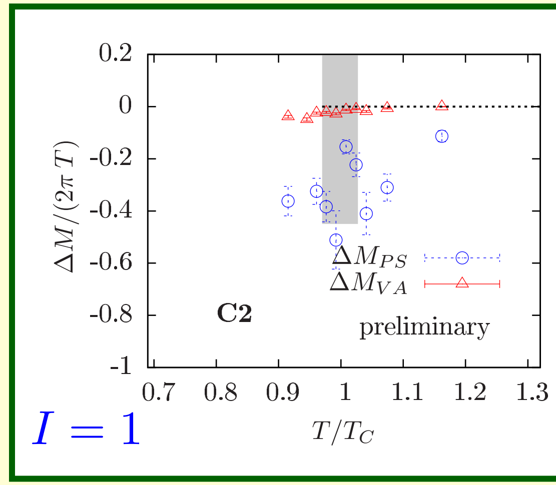

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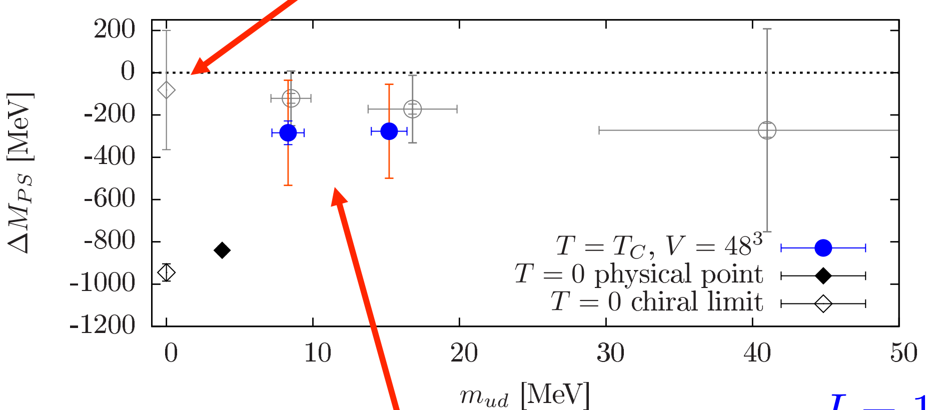
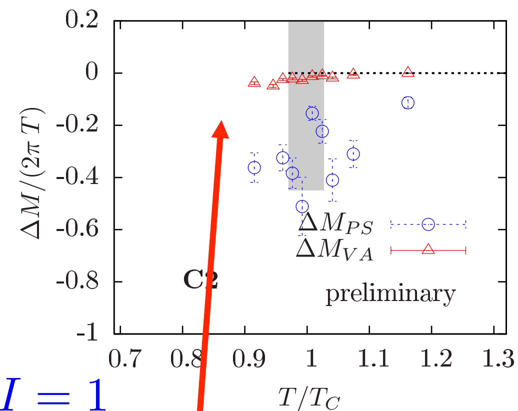
Compatible with $U(1)_A$ restoration @T_c in chiral limit



$I = 1$



$I = 1$

Small $U(1)_A$ breaking in phys.limit, increasing with larger volumes

ρ/a_1 $O(4)$
more efficient

WARD IDENTITIES

AGN, J.Ruiz de Elvira, 2016, 2018

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$$\boxed{\chi_P^{ls}(T) = -2\frac{m_l}{m_s}\chi_{5,disc}(T) = -\frac{2}{m_l m_s}\chi_{top}(T)}$$

$\chi_{5,disc} = \frac{1}{4}\left(\chi_P^{\pi} - \chi_P^{\eta_l}\right)$ **measures** $O(4)\times U(1)_A$ **restoration**

$\chi_{top} = -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x) A(0)\rangle$ **topological susceptibility**, $A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

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$(*)\ \eta_l \to i\bar{\psi}_l \gamma_5 e^{i\frac{\pi}{2}\gamma_5 \tau^b} \psi_l = -\delta^b$

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Also Azcoiti 2016 with similar WI

$\Longrightarrow O(4) \times U(1)_A$ **pattern at *exact* chiral restoration**

(hence consistent with Cossu, Aoki, Brandt et al $N_f = 2$, $m_l \to 0$)

efficient $\eta_l - \delta$ degeneration $@T_c$ (much worse in 2+1)

$I = 1/2$ **SECTOR** (K/κ)

- $K^a = i\bar{\psi}\gamma_5\lambda^a\psi \leftrightarrow \kappa^a = \bar{\psi}\lambda^a\psi$ **degenerate under *both*** $O(4)$ **and** $U(1)_A$

  $(\psi^T = (u,d,s), a = 4,\ldots 7)$ **(lowest states** K **and** $K_0^*(700)/\kappa$**)**

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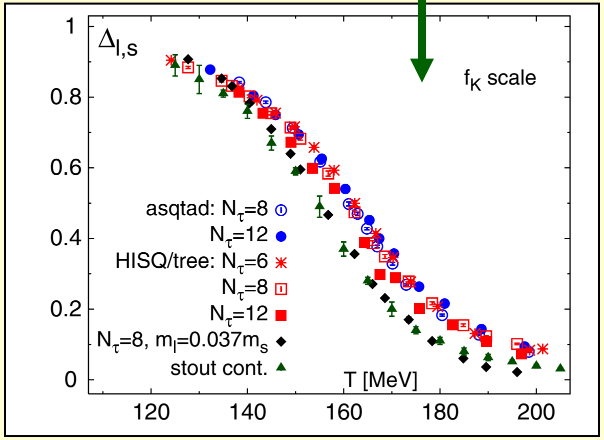
From WIs:

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2m_s}{m_s^2 - m_l^2}\left[\underbrace{\langle\bar{q}q\rangle_l(T) - 2\frac{m_l}{m_s}\langle\bar{s}s\rangle(T)}_{\text{dictated by subtracted condensate } \Delta_{l,s}}\right]$$



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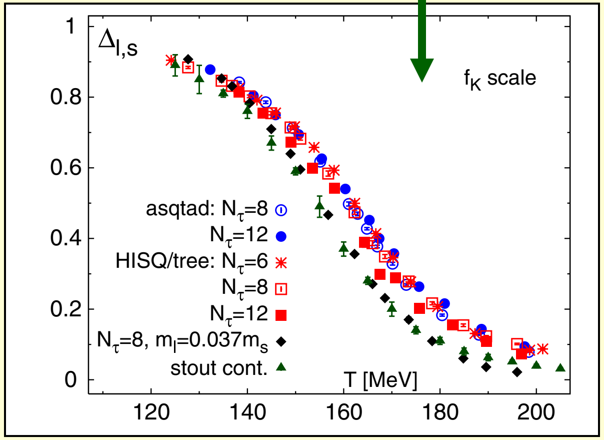


$\Rightarrow$ **In $N_f = 2$ limit, exact $O(4)\times U(1)_A$ degen. for $m_l, \langle\bar{q}q\rangle_l \to 0^+$**

$\Rightarrow$ **In physical case strength of $U(1)_A$ above T_c well determined and driven by $\langle\bar{s}s\rangle$**

$\Rightarrow$ **May help to clarify the role of strangeness (in progress)**

EFFECTIVE THEORIES

- **Effective Theories needed for systematic analysis below the transition in terms of physical hadrons.**

- **SU(3) and U(3) ChPT model-independent framework for light mesons (π, K, η, η') within $1/N_c \sim m_q \sim T^2 \sim p^2$ counting.** (1)

- **Light meson scattering dominant interactions in the thermal bath. Unitarized scattering generates (thermal) resonances** (2)

- **HRG approach includes heavier states and describes well (monotonically) most observables for $T \lesssim T_c$** (3)

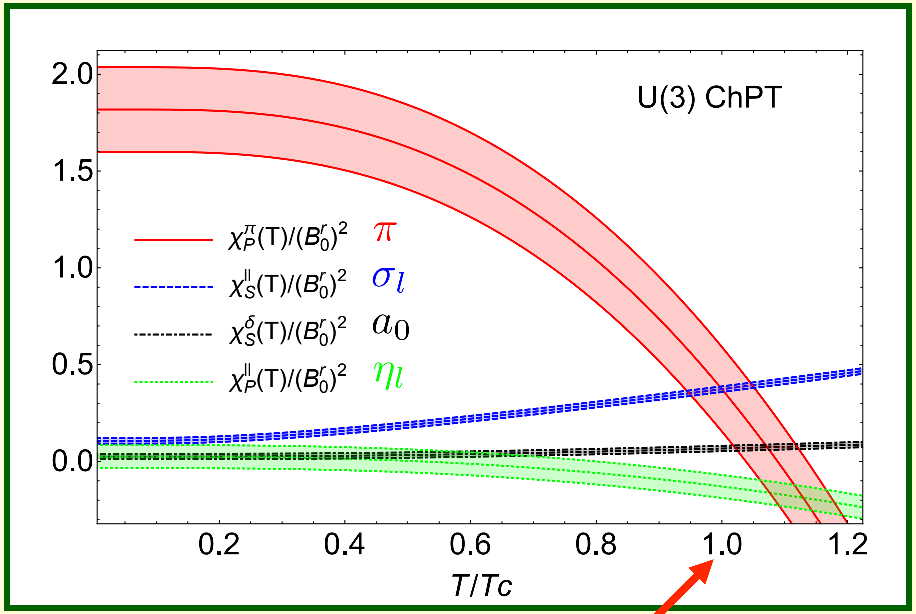
- **Relevant observables where (U)ChPT useful for CSR** $>>>$

(1) Gasser, Leutwyler, Gerber, Kaiser, Herrera-Siklody et al, AGN, Ruiz de Elvira, ...

(2) Cabrera, Dobado, Fernández-Fraile, AGN, Llanes-Estrada, Peláez, Ruiz de Elvira, Torres-Andrés, Vioque, $\cdots$

(3) Karsch, Tawfik, Redlich, Tawfik-Toublan, Huovinen, Petreczcky, Jankowski, Blaschke, Spalinski, ...

Partners in $U(3)$ ChPT



Differences within ChPT uncertainty in massive case.
(degeneration $T_{U(1)_A} \sim 1.1 T_{chiral}$)

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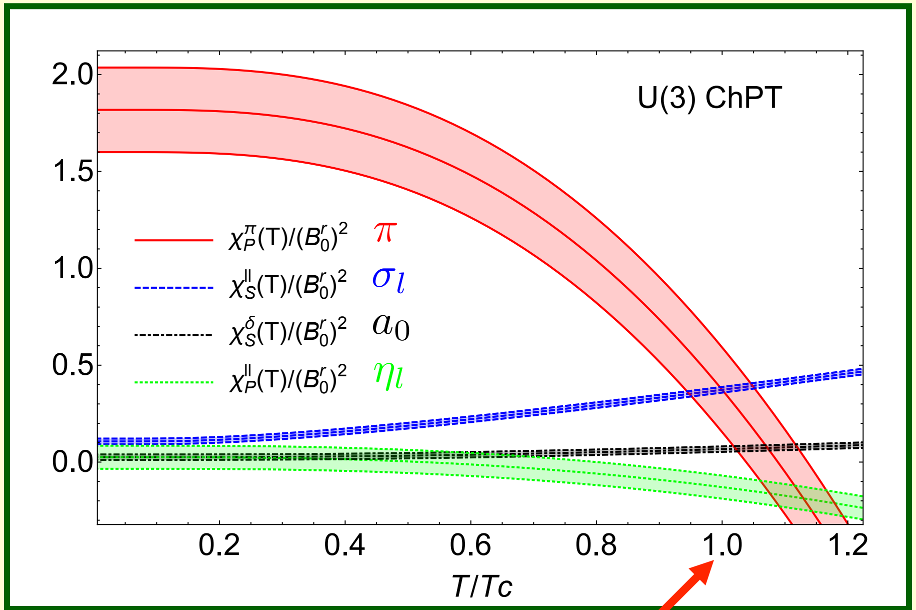

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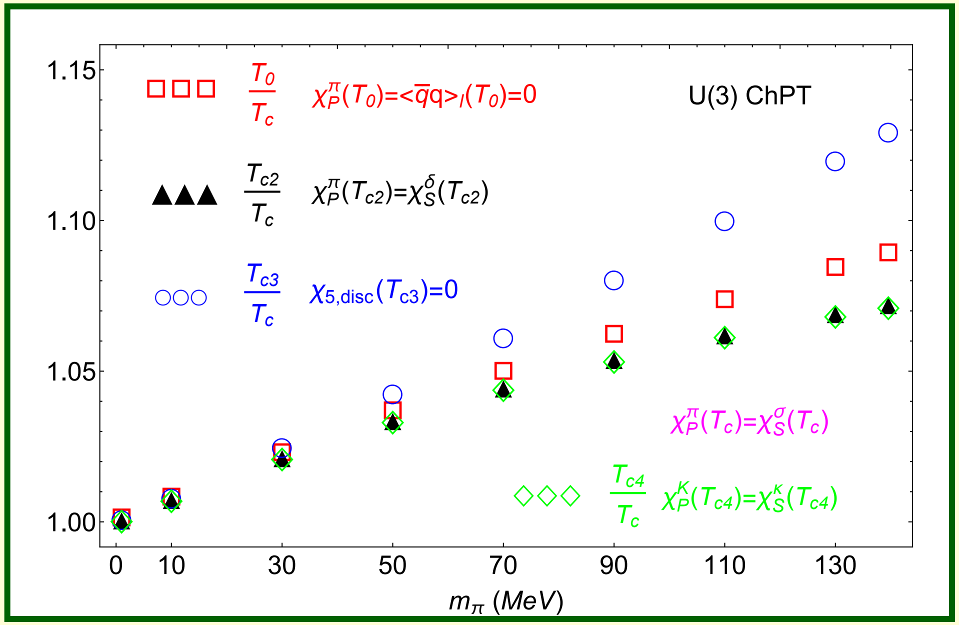

$\rightarrow O(4)\times U_A(1)$ in chiral limit

Disconnected susceptibility and $O(4) \times U(1)_A$ restoration

$$\boxed{\chi_S^\sigma = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^\delta + 4\chi_S^{dis}} \quad (m_u = m_d)$$

On the other hand,

$$\boxed{\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^\pi(T) - \chi_S^\sigma(T)\right] + \frac{1}{4}\left[\chi_S^\delta(T) - \chi_P^{\eta_l}(T)\right]}$$

$\Rightarrow$ **Is the vanishing of $\chi_{5,disc}$ in conflict with χ_S^{dis} peaking at the chiral transition?**

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From ChPT in the chiral limit $M_\pi \to 0^+$(IR),

$$\Rightarrow \chi_{5,disc}(T_c) \sim \mathcal{O}\left(\frac{T_c}{M_\pi}\right) \sim \chi_S^{dis}(T_c)$$

"peak" with same coeff.

$$\chi_{5,disc}(T_c) = \chi_S^{dis}(T_c) + \frac{1}{4}\cancel{\left[\chi_P^\pi(T_c) - \chi_S^\sigma(T_c)\right]} + \underbrace{\frac{1}{4}\left[\chi_S^\delta(T_c) - \chi_P^{\eta_l}(T_c)\right]}_{IR\ regular}$$

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$$\boxed{\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^\pi(T) - \chi_S^\sigma(T)\right] + \frac{1}{4}\left[\chi_S^\delta(T) - \chi_P^{\eta_l}(T)\right]}$$

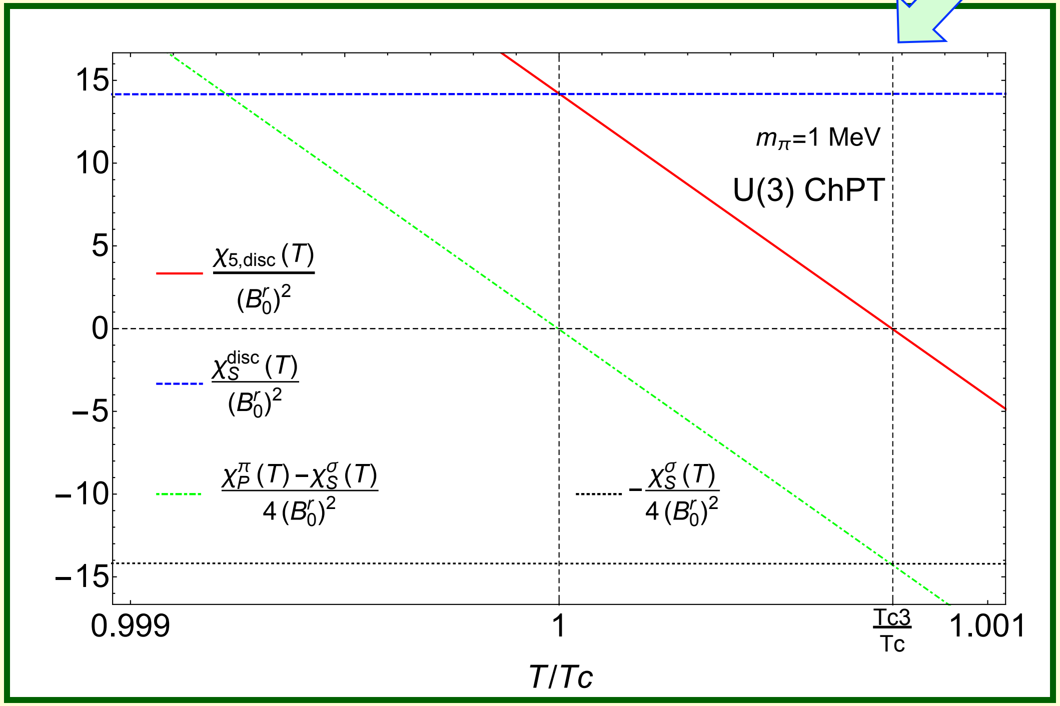
From ChPT in the chiral limit $M_\pi \to 0^+$(IR), $\Rightarrow T_{c3} = T_c + \mathcal{O}\left(M_\pi^2\right)$

$$\Rightarrow \chi_S^{disc}(T_{c3}) \sim \mathcal{O}\left(\frac{T_{c3}}{M_\pi}\right) \sim \frac{1}{4}\chi_S^\sigma(T_{c3}) \quad \text{"peak" with same coff.}$$

$$\chi_{5,disc}(T_{c3}) = 0 = \chi_S^{dis}(T_{c3}) + \frac{1}{4}\left[\cancel{\chi_P^\pi(T_{c3})} - \chi_S^\sigma(T_{c3})\right] + \frac{1}{4}\left[\underbrace{\chi_S^\delta(T_{c3})}_{IR\ regular} - \cancel{\chi_P^{\eta_l}(T_{c3})}\right]$$

Disconnected susceptibility and $O(4) \times U(1)_A$ restoration

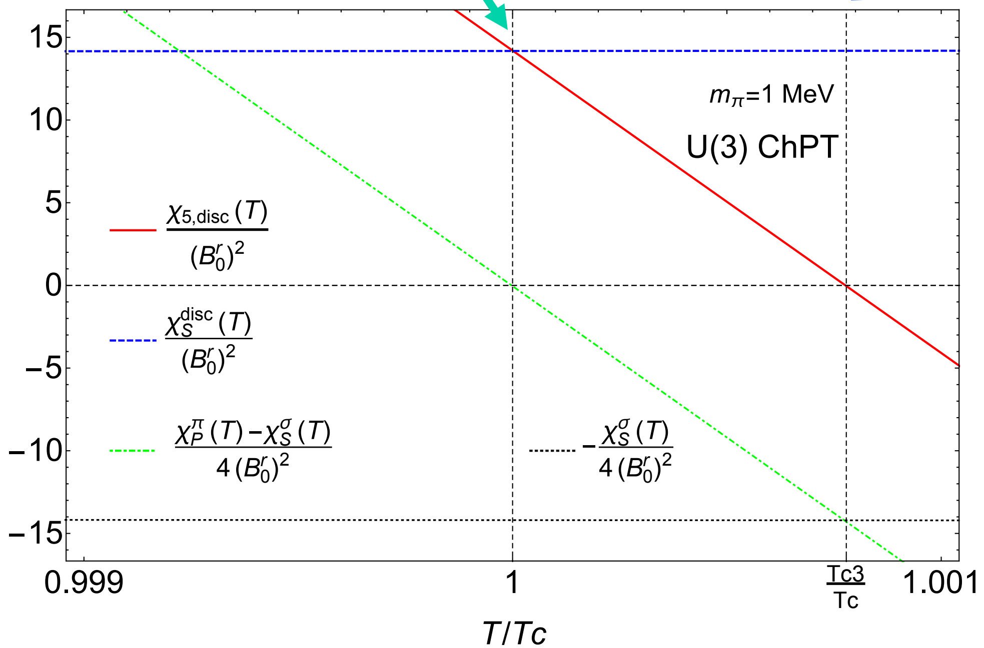
Vanishing of $\chi_{5,dis}$ at $O(4) \times U(1)_A$ compatible with χ_S^{dis} peak for $m_l \to 0^+$



Disconnected susceptibility and $O(4) \times U(1)_A$ restoration

Both peak at chiral T_c

Vanishing of $\chi_{5,dis}$ at $O(4) \times U(1)_A$ compatible with χ_S^{dis} peak for $m_l \to 0^+$



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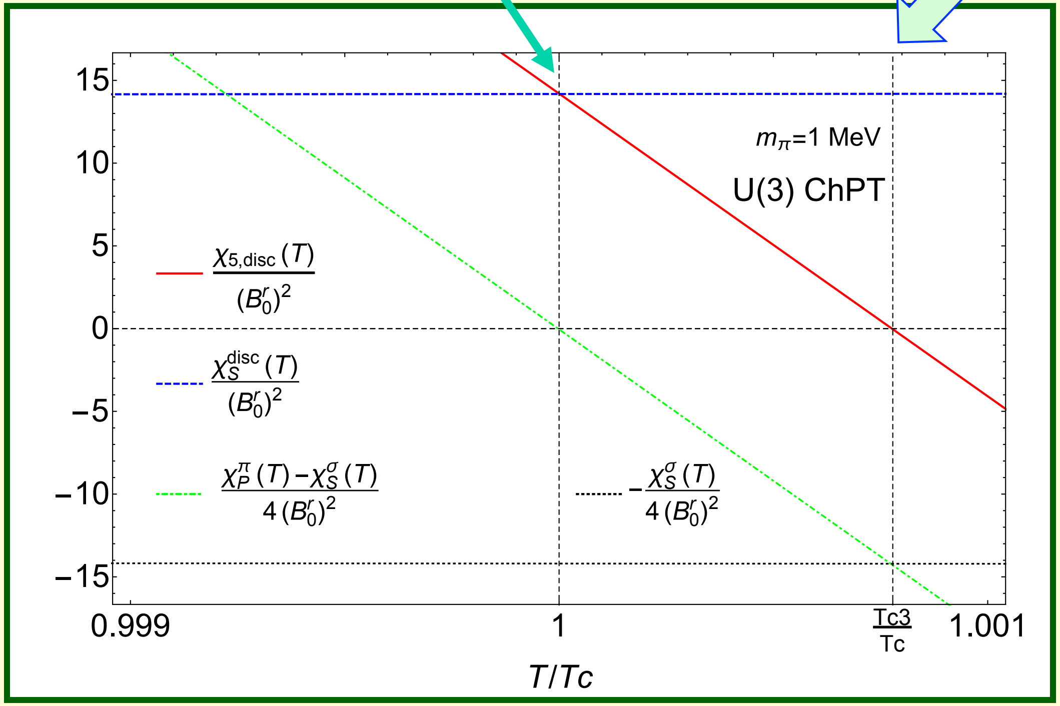
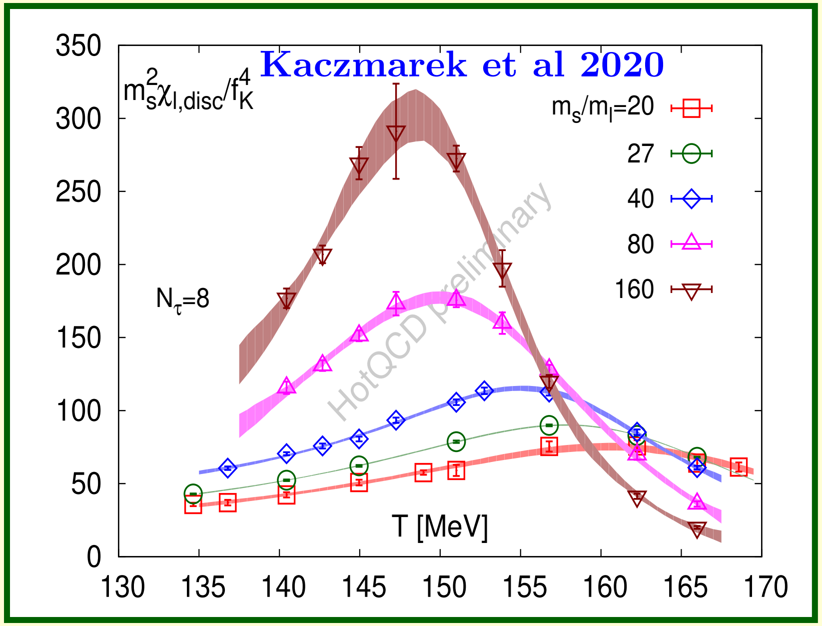




$\Rightarrow$ χ_S^{dis} **may not be the best $O(4) \times U(1)_A$ indicator near chiral limit**

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, 2018

$\Rightarrow$ χ_S **saturated by lightest scalar pole: thermal $f_0(500)$**

$$\boxed{\chi_S(T) \simeq \chi_S(0)\frac{M_S^2(0)}{M_S^2(T)}}$$

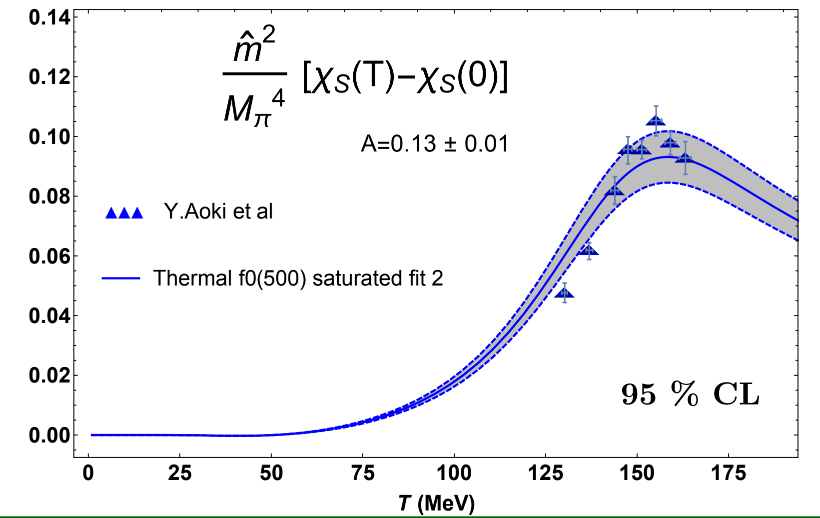
$$M_S^2(T) = \mathbf{Re}(s_{pole}(T)) \sim \mathbf{Re}\Sigma_{f_0}$$

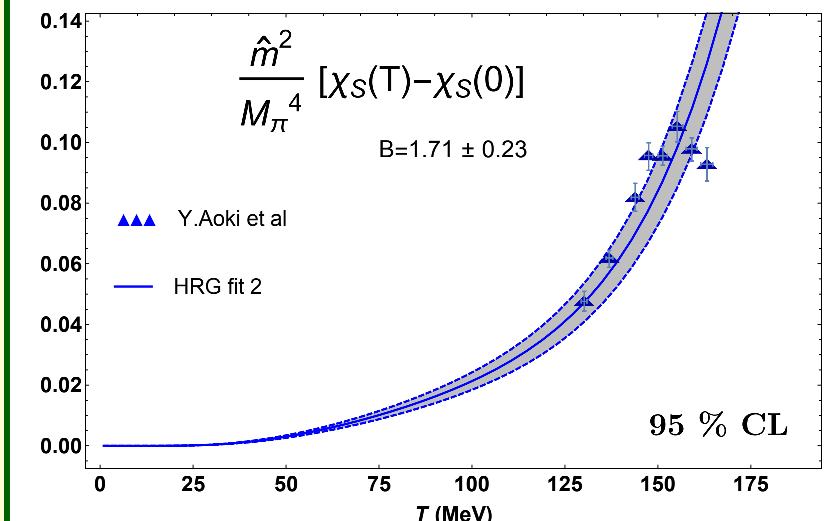
calculated from UChPT $\pi\pi$ scattering at $T \neq 0$

$\Rightarrow$ **Same scaling expected for screening masses if they don't differ much from pole masses around T_c (OK with lattice through WI)**

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, 2018



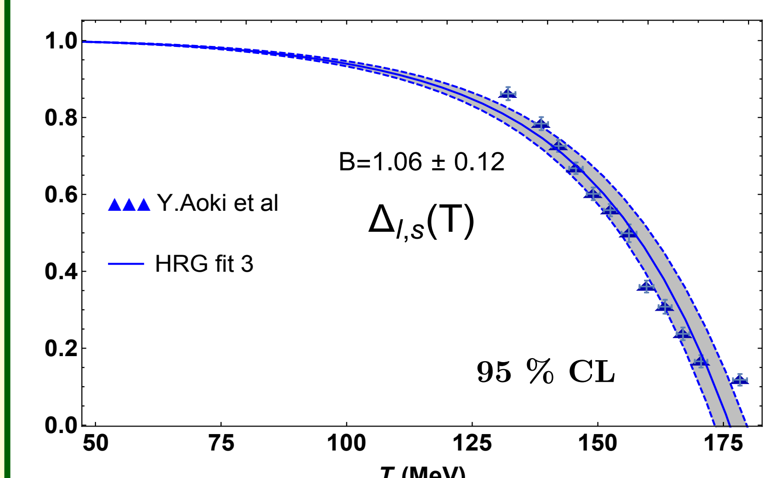


$$\chi_S(T) = A\frac{m_\pi^4}{4m_q^2}\frac{M_S^2(0)}{M_S^2(T)} \qquad (A_{ChPT} \simeq 0.14)$$

HRG Jankowski et al 2013

free energy density normalization B fitted.

Fit	A	B	χ^2/dof	T_{max}(MeV)
Thermal f_0 fit 1	0.13± 0.02	—	6.25	155
Thermal f_0 fit 2	0.13± 0.01	—	4.93	165
HRG fit 1	—	1.90± 0.02	1.33	155
HRG fit 2	—	1.71± 0.23	10.30	165
HRG fit 3	—	1.06± 0.12	3.77	155



- **Thermal f_0 approach better around T_c**
- **HRG fits of $\Delta_{l,s}$ and χ_S at conflict**

Topological Charge in $U(3)$ ChPT

AGN, J.R.Elvira, A.Vioque, 2019

$$\epsilon_{vac}(\theta) = \epsilon_{vac}(0) + \frac{1}{2}\chi_{top}\theta^2 + \frac{1}{24}c_4\theta^4 + \dots$$

$\sim$ **Axion mass** $\quad\sim$ **Axion coupling**

$$\chi_{top}^{U(3),LO} = \Sigma \frac{M_0^2 \bar{m}}{M_0^2 + 6B_0\bar{m}} \qquad c_4^{U(3),LO} = -\frac{\Sigma}{\bar{m}^{[3]}}\left(\frac{M_0^2\bar{m}}{M_0^2 + 6B_0\bar{m}}\right)^4$$

$$\bar{m} = \left[\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}\right]^{-1} \qquad \bar{m}^{[3]} = \left[\frac{1}{m_u^3} + \frac{1}{m_d^3} + \frac{1}{m_s^3}\right]^{-1}$$

$(-\Sigma)$ LO quark condensate $\qquad M_0$ anomalous part of $M_{\eta'}$ $(m_{u,d,s} = 0)$

- **vanish** $m_q \to 0 \Rightarrow$ **expected to be well described within ChPT**
- $SU(3)$ **for** $M_0 \to \infty$, $SU(2)$ **for** $M_0, m_s \to \infty$
- **Quenched** $m_q \to \infty$: $\chi_{top}^{LO} = F^2 M_0^2/6$

  (Witten-Veneziano 1979) $\to$ **meson loops crucial**

Leutwyler,Smilga 1992: $SU(3)$ **LO**
Mao et al 2009; Bernard et al: 2012: $SU(3)$ **NLO**
Grilli et al 2016: $T \neq 0$ $SU(2)$ **NLO**

Topological Charge in $U(3)$ ChPT

AGN, J.R.Elvira, A.Vioque, 2019

$\rightarrow$ $T = 0$ **results** $(m_u = m_d)$:

$\chi_{top}^{1/4}$ [MeV]	U(3)	SU(2)	SU(3)
LO	74(3)	75(3)	75(3)
NLO	74(3)	78(3)	83(2)
NNLO	81(2)		

$[\chi_{top}^{latt}]^{1/4} = 73(9)$ **(Bonati et al 2016)**

$b_2 = \frac{c_4}{12\chi_{top}}$	U(3)	SU(2)	SU(3)
LO	-0.01737(4)	-0.02083	-0.01960
NLO	-0.018(2)	-0.029(2)	-0.025(1)
NNLO	-0.023(2)		

$b_2^{latt} = -0.0216(15)$ **(Bonati et al 2016, gluodynamics)**

$\Rightarrow$ $SU(2)$ **dominates,** $U(3)$ η' **loops and** $\eta - \eta'$ **mixing of the same order than** $SU(3)$ K, η **loops**

$\Rightarrow$ **full** $U(3)$ **in agreement with lattice within uncertainties (LEC and lattice, larger lattice uncertainty for** b_2**)**

$\Rightarrow$ **In addition, within** $U(3)$ **ChPT, explicit expressions for the leading and subleading** $1/N_c$ **contributions can be obtained:**

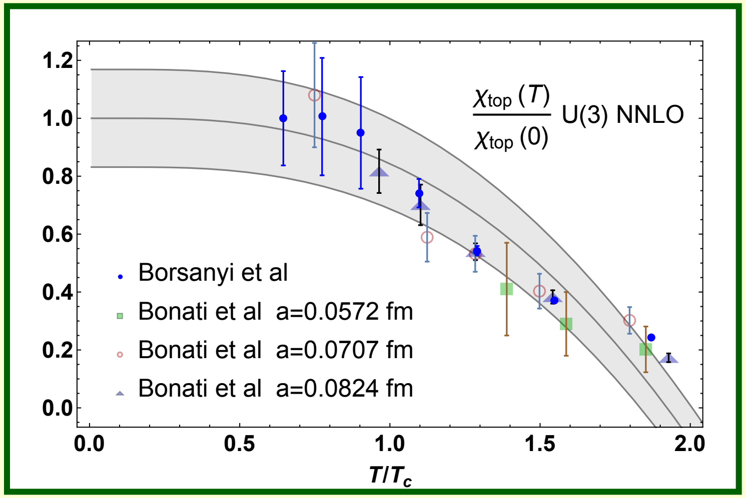
$$\chi_{top} = \underbrace{\mathcal{O}(1)}_{\text{Witten-Veneziano term}} + \mathcal{O}(N_c^{-1}) + \ldots$$

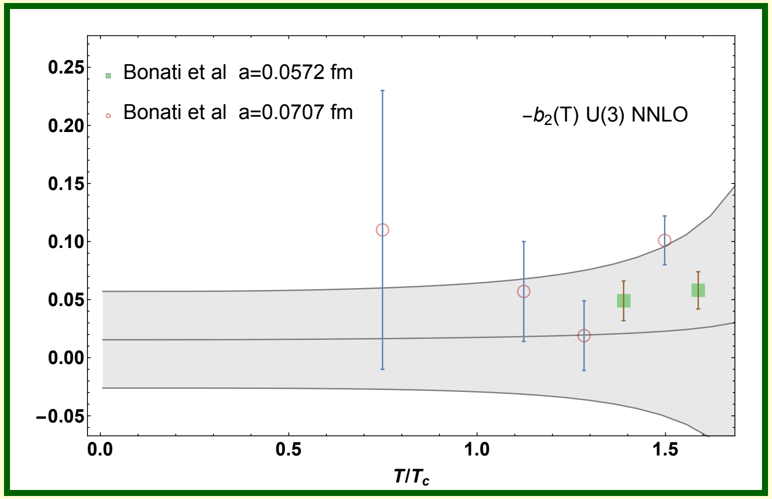
$$c_4 = \underbrace{\mathcal{O}(N_c^{-2})}_{\text{constant } \theta^4 \text{ term in } \mathcal{L}} + \mathcal{O}(N_c^{-3}) + \ldots$$

OK with lattice (Vicari, Panagopoulos, Bonati et al)

Topological Charge in $U(3)$ ChPT

AGN, J.R.Elvira, A.Vioque, 2019





- T-dependence captured by ChPT within uncertainties (larger for c_4) far beyond the low-T applicability regime

- Scales with T dominated by $\langle \bar{q}q \rangle_l^{ChPT}$ at low T
- However, 2nd term in WI $\chi_{top} = -\frac{1}{4}\left[m_{ud}\langle \bar{q}q \rangle_l + m_{ud}^2 \chi^{\eta_l}\right]$ relevant near T_c

  $\rightarrow$ finite $O(4)$-$U(1)_A$ gap in physical case

Chiral Imbalance in ChPT

D.Espriu, AGN, A.Vioque, 2020

- μ_5 **chem. pot. for approx. (local) conservation of chiral charge** Q_5 **(characteristic time** $t_{L-R} \sim m_q^{-1} \gg t_{fireball}$ **for light quarks)**

  $\Rightarrow \mathcal{L}_{QCD} \to \mathcal{L}_{QCD} + \mu_5 \bar{\psi}\gamma^0\gamma_5\psi$ **in that region**

$$Q_5 = \int_{vol} d^3\vec{x}\ \bar{\psi}\gamma^0\gamma_5\psi$$

- **GOAL: Construct the most general meson eff. lagr. for** $\mu_5 \neq 0$

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$$Q_5 = \int_{vol} d^3\vec{x}\ \bar{\psi}\gamma^0\gamma_5\psi$$

- **GOAL: Construct the most general meson eff. lagr. for** $\mu_5 \neq 0$

- **"local" chiral invariant ChPT with axial** $U(1)$ **external source**

D_μ **& explicit source-dep new terms**

$N_f = 2$

$Z \sim 0.8$ **(EM pion mass dif)**

$$\mathcal{L}_2 \to \mathcal{L}_2 + 2\mu_5^2 F^2 (1 - Z + \kappa_0)$$

κ_i **new LEC to be fixed**

$$\mathcal{L}_4 \to \mathcal{L}_4 + \kappa_1 \mu_5^2 \mathbf{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \kappa_2 \mu_5^2 \mathbf{tr}\left(\partial_0 U^\dagger \partial^0 U\right) + \kappa_3 M_\pi^2 \mu_5^2 \mathbf{tr}\left(U + U^\dagger\right) + \kappa_4 \mu_5^4$$

- **NLO dispersion relation** $\Longrightarrow$

No lattice data

$$v_\pi(\mu_5) = \left.\frac{|\vec{p}|}{p_0}\right|_{M=0} = 1 + 2\kappa_2 \frac{\mu_5^2}{F_\pi^2} \qquad (\kappa_2 < 0)$$

$$\left[M_\pi^2\right]^{pole}(\mu_5) = M_\pi^2\left[1 - 4(\kappa_1+\kappa_2-\kappa_3)\frac{\mu_5^2}{F^2}\right]$$

$$\left[M_\pi^2\right]^{scr}(\mu_5) = M_\pi^2\left[1 - 4(\kappa_1-\kappa_3)\frac{\mu_5^2}{F^2}\right]$$

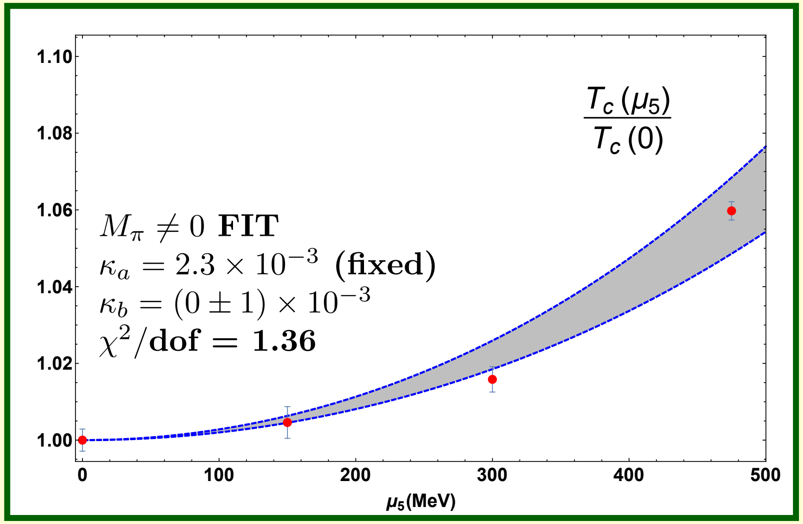
- **NNLO quark condensate** $\Longrightarrow$

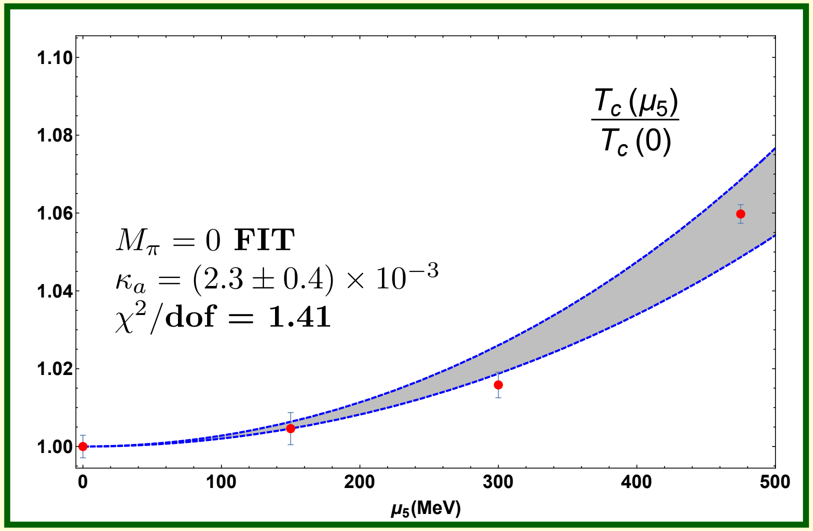
$$\left.\frac{\langle \bar{q}q\rangle_l (T,\mu_5)}{\langle \bar{q}q\rangle_l (0,\mu_5)}\right|_{M=0} = 1 - \frac{T^2}{8F^2}\left[1 - 2\kappa_a \frac{\mu_5^2}{F^2}\right] - \frac{T^4}{384F^4}$$

$$\kappa_a = 2\kappa_1 - \kappa_2, \qquad \kappa_b = \kappa_1 + \kappa_2 - \kappa_3$$

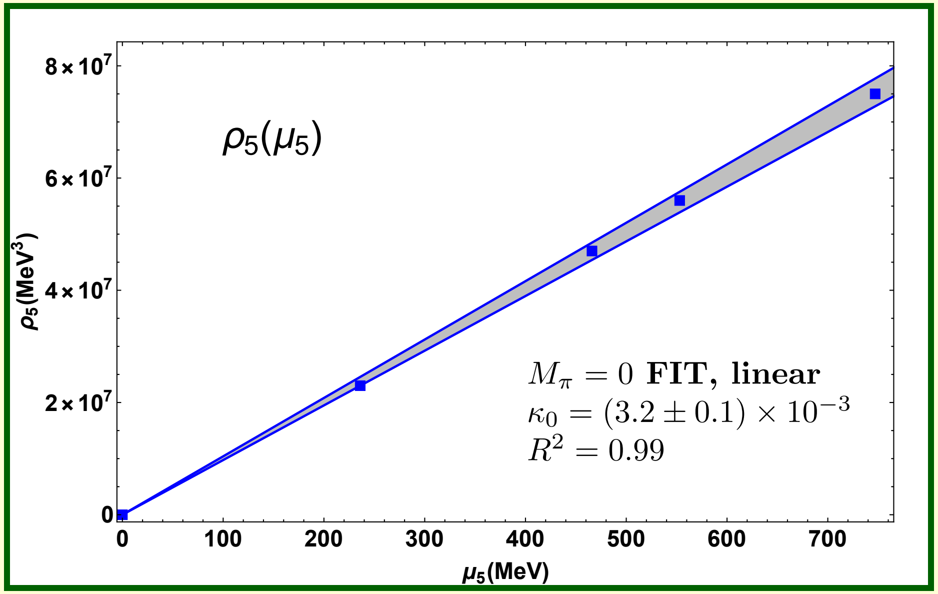
Well acommodated by chiral limit ChPT:

Lattice data Braguta et al 2015 ($N_c = 2$)





- **NLO chiral charge density** $\Longrightarrow$



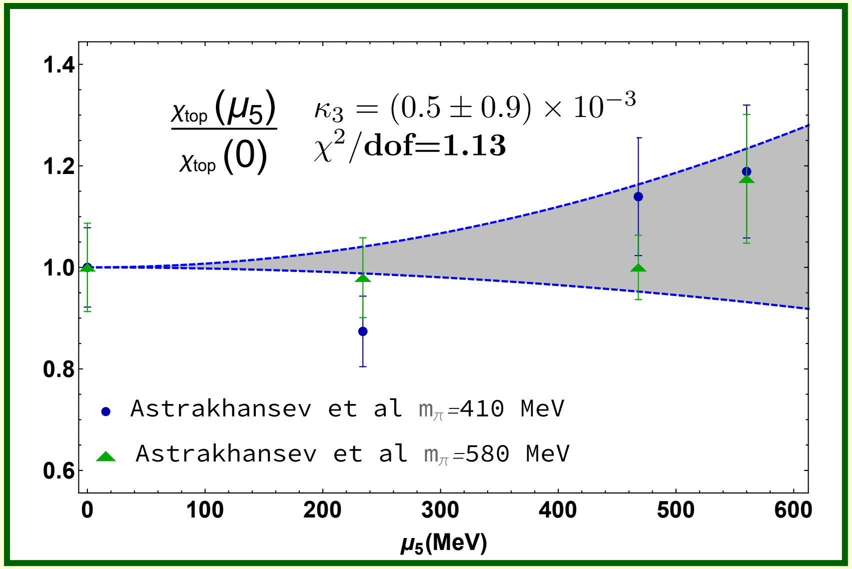
Linear $\mathcal{O}(\mu_5)$ dominant for moderate μ_5

Very insensitive to M_π

Lattice data Astrakhantsev et al 2019

- **NLO topological susceptibility** $\Longrightarrow$

$$\frac{\chi_{top}(\mu_5)}{\chi_{top}(0)} = 1 + 4\frac{\kappa_3 \mu_5^2}{F^2}$$



More lattice data needed to better pin down κ_i

Lattice data Astrakhantsev et al 2019

CONCLUSIONS

- $\star$ **Nature of chiral transition and interplay with $U(1)_A$, keys for QCD phase diagram**

- $\star$ **Stronger $U(1)_A$ breaking @T_c for $N_f = 2+1$**

  (role of strangeness and chiral limit to be understood)

- $\star$ **WI $\Rightarrow$ $O(4) \times U(1)_A$ for exact chiral restoration of S/P nonet**

  OK with $N_f = 2$ lattice

- $\star$ **WI $\Rightarrow K/\kappa$ suitable channel $\rightarrow$ $O(4) \times U(1)_A \sim \Delta_{l,s}$ driven by $\langle \bar{s}s \rangle$**

- $\star$ **Eff.Th. $\Rightarrow$ Patterns and partners OK with WI**

  Saturated χ_S with thermal $f_0(500)$ OK with lattice

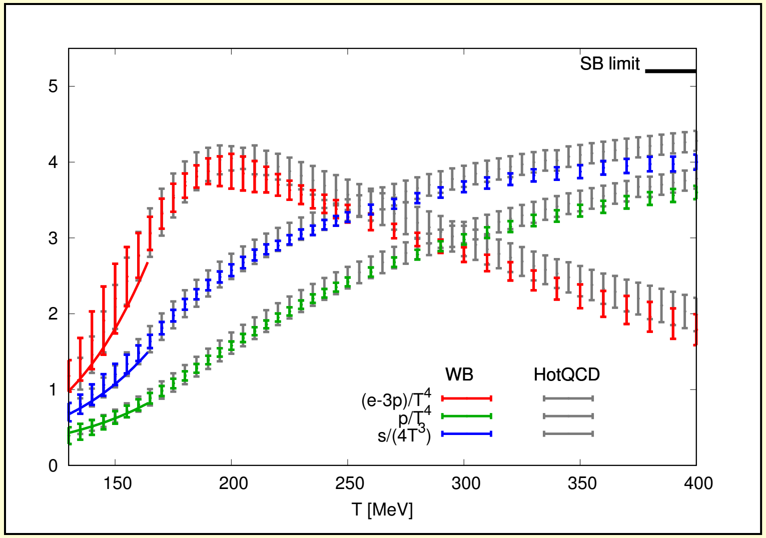
  χ_{top}, c_4 well described by $U(3)$ ChPT

  $\mu_5 \neq 0$ ChPT $\rightarrow$ OK for $\mu_5 \lesssim$ 600 MeV incl.$T \neq 0$

BACKUP SLIDES

OTHER HIGHLIGHTS OF QCD TRANSITION

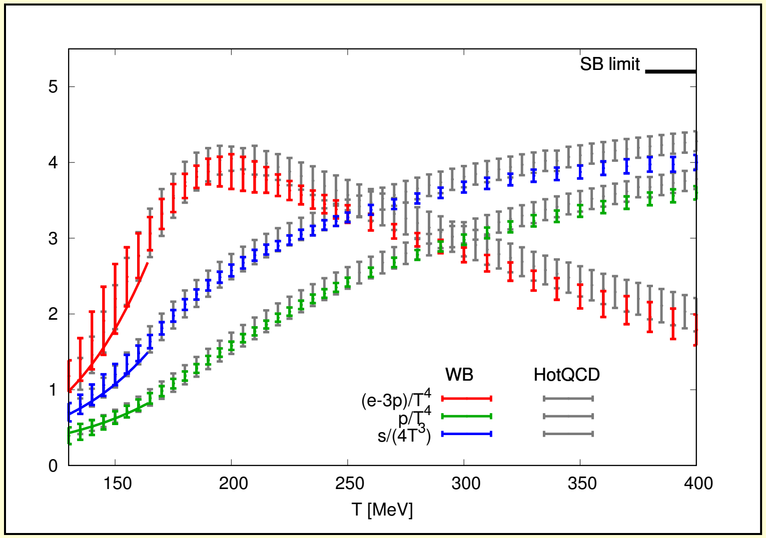
Pressure, entropy, trace anomaly



From C.Ratti 2018 (2014 WB, HotQCD data)

OTHER HIGHLIGHTS OF QCD TRANSITION

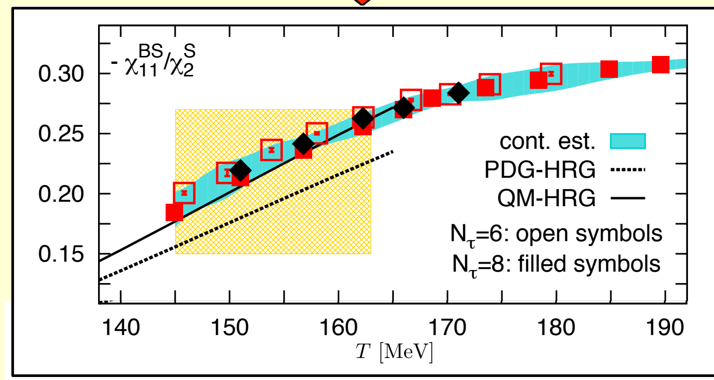
Pressure, entropy, trace anomaly



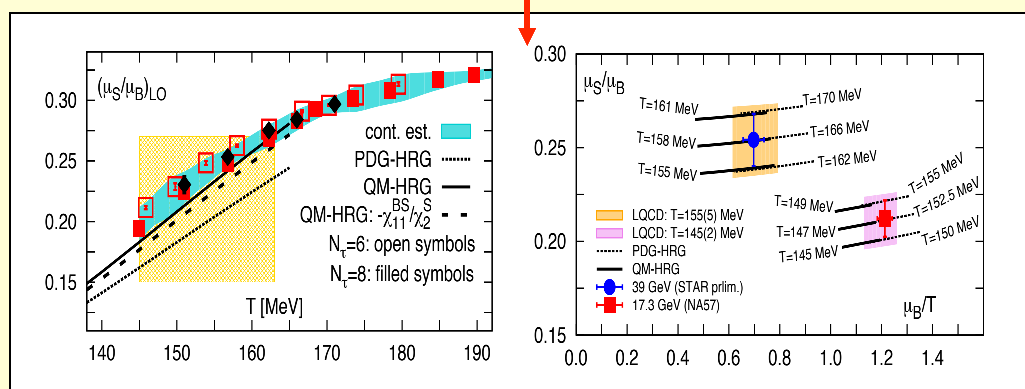
From C.Ratti 2018 (2014 WB, HotQCD data)

Fluctuations of conserved charges (Q,B,S)

Bazavov et al 2014



related to strangeness freeze-out conditions



$$n_S = 0,\; n_Q/n_B = 0.4$$

CHIRAL PATTERNS AND PARTNERS

$\Rightarrow$ **Is $U(1)_A$ asymptotic restoration effective already at T_c?**
(specially near chiral limit)

- **Chiral limit behaviour still under debate**

- **$U(1)_A$ @T_c affects also critical point at $\mu_B \neq 0$** Mitter, Schaefer 2014

- **$U(1)_A$ restoration shows in $M_{\eta'}$ reduction in effective theories, lattice and experiment (increase of η' production in dileptons&photons)**

  Ishii et al 2017. Gu et al 2018. AGN, J.R.Elvira 2018. Kotov, Lombardo et al 2019

  Kapusta, Kharzeev, McLerran 1996. Csorgo, Vertesi, Sziklai 2010

Ward Identities

Formally from QCD through A/V transformations:

$$\left\langle \frac{\delta \mathcal{O}_P(y)}{\delta \alpha_A^a(x)} \right\rangle = - \left\langle \mathcal{O}_P(y) \bar{\psi}(x) \left\{ \frac{\lambda^a}{2}, \mathcal{M} \right\} \gamma_5 \psi(x) \right\rangle + i \frac{\delta_{a0}}{\sqrt{6}} \langle \mathcal{O}_P(y) A(x) \rangle$$

$$\left\langle \frac{\delta \mathcal{O}_S(y)}{\delta \alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}_S(y) \bar{\psi}(x) \left[\frac{\lambda^a}{2}, \mathcal{M} \right] \psi(x) \right\rangle$$

$$\lambda^0 = \sqrt{2/3}\, \mathbb{1}, \quad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu} \tilde{G}^{\mu\nu}$$

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$\lambda^0 = \sqrt{2/3}\,\mathbb{1}$, $A(x) = \frac{3\alpha_s}{4\pi}Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

$\mathcal{O}_P^b = i\bar{\psi}\gamma_5\lambda^b\psi \equiv P^b \rightarrow$ **1p vs 2p fns** $\rightarrow \langle\bar{q}q\rangle$ **vs** χ_P

$\mathcal{O}_P^{bc} = P^b S^c \rightarrow$ **2p vs 3p** $\rightarrow$ **ch.partners vs meson vertices**
(e.g. $\chi^\sigma - \chi^\pi \sim \sigma\pi\pi, \ldots$**)**

$\mathcal{O}_S^b = \bar{\psi}\lambda^b\psi \equiv S^b \rightarrow \langle\bar{q}q\rangle$ **vs** χ_S **for** κ **sector** $b = 4,\ldots,7$

WARD IDENTITIES obtained from the QCD generating functional may shed light on chiral patterns and partners

AGN, J.Ruiz de Elvira, 2016, 2018

- π **SECTOR** $\rightarrow \langle\bar{q}q\rangle_l(T) = -\hat{m}\chi_P^\pi(T)$
- K **SECTOR** $\rightarrow \langle\bar{q}q\rangle_l(T) + 2\langle\bar{s}s\rangle(T) = -(\hat{m}+m_s)\chi_P^K(T)$
- η, A **SECTOR** $\rightarrow$ η/η' **mixing &** $U_A(1)$ **anomaly enter:**

$$\begin{aligned}
\chi_P^{\eta_l}(T) &= -\frac{\langle\bar{q}q\rangle_l(T)}{\hat{m}} - \frac{4}{\hat{m}^2}\chi_{top}(T)\\
\chi_P^{\eta_s}(T) &= -\frac{\langle\bar{s}s\rangle(T)}{m_s} - \frac{1}{m_s^2}\chi_{top}(T)\\
\chi_P^{ls} &= -\frac{\hat{m}}{2m_s}\left[\chi_P^\pi(T) - \chi_P^{\eta_l}(T)\right] = -\frac{2}{\hat{m}m_s}\chi_{top}(T)
\end{aligned}$$

$\chi_P^{ab} \equiv \int_T dx\,\langle P^a(x)P^b(0)\rangle,\ \langle\bar{q}q\rangle_l = \langle\bar{u}u + \bar{d}d\rangle,\ \hat{m} = m_u = m_d$ $\qquad A(x) = \frac{3\alpha_s}{4\pi}Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

$\chi_{top} \equiv -\frac{1}{36}\int_T dx\langle\mathcal{T}A(x)A(0)\rangle$ **TOPOLOGICAL SUSCEPTIBILITY**

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$O(4)$ **partners**

$\sigma\pi\pi$ **vertex**

$\rightarrow \pi\pi$ **scattering** $I = J = 0$

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \sigma_l(y)\pi(x)\pi(0) \rangle$$

$$P_{ll}(y) - S_{\delta\delta}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \delta(y)\pi(x)\eta_l(0) \rangle$$

$$P_{ls}(y) = \frac{1}{3}\hat{m} \int_T dx \, \langle \mathcal{T} \eta_s(y)\pi(x)\delta(0) \rangle$$

$$S_{ls}(y) = -\frac{1}{3}\hat{m} \int_T dx \, \langle \mathcal{T} \sigma_s(y)\pi(x)\pi(0) \rangle$$

$$d^{abc} \left[P_{KK}(y) - S_{\kappa\kappa}(y) \right] = \hat{m} \int_T dx \, \left\langle \mathcal{T} K^b(y)\kappa^c(x)\pi^a(0) \right\rangle$$

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$U(1)_A$ **partners**

$$
\begin{aligned}
P_{\pi\pi}(y) - S_{\delta\delta}(y) &= \int_T dx\, \langle \mathcal{T} \pi(y)\delta(0)\tilde{\eta}(x)\rangle \\
P_{ll}(y) - S_{ll}(y) &= \int_T dx\, \langle \mathcal{T} \eta_l(y)\sigma_l(0)\tilde{\eta}(x)\rangle \\
P_{ls}(y) - S_{ls}(y) &= \int_T dx\, \langle \mathcal{T} \eta_l(y)\sigma_s(0)\tilde{\eta}(x)\rangle \\
P_{ss}(y) - S_{ss}(y) &= \int_T dx\, \langle \mathcal{T} \eta_s(y)\sigma_s(0)\tilde{\eta}(x)\rangle \\
P_{KK}(y) - S_{\kappa\kappa}(y) &= \int_T dx\, \langle \mathcal{T} K(y)\kappa(0)\tilde{\eta}(x)\rangle
\end{aligned}
$$

$\tilde{\eta}(x) = \hat{m}\eta_l(x) + m_s\eta_s(x) + \frac{1}{2}A(x)$ **three sources of** $U(1)_A$ **breaking**

WI AND LATTICE SCREENING MASSES

Assuming soft T behavior for residues and M_{sc}/M_{pole} of correlators $K_{P,S}$:

$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \rightarrow$ **measured in lattice**

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$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \rightarrow$ **measured in lattice**

$$\frac{M_\pi^{sc}(T)}{M_\pi^{sc}(0)} \sim \left[\frac{\chi_P^\pi(0)}{\chi_P^\pi(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)}{\langle\bar{q}q\rangle_l(T)}\right]^{1/2}$$

$$\frac{M_K^{sc}(T)}{M_K^{sc}(0)} \sim \left[\frac{\chi_P^K(0)}{\chi_P^K(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)+2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T)+2\langle\bar{s}s\rangle(T)}\right]^{1/2}$$

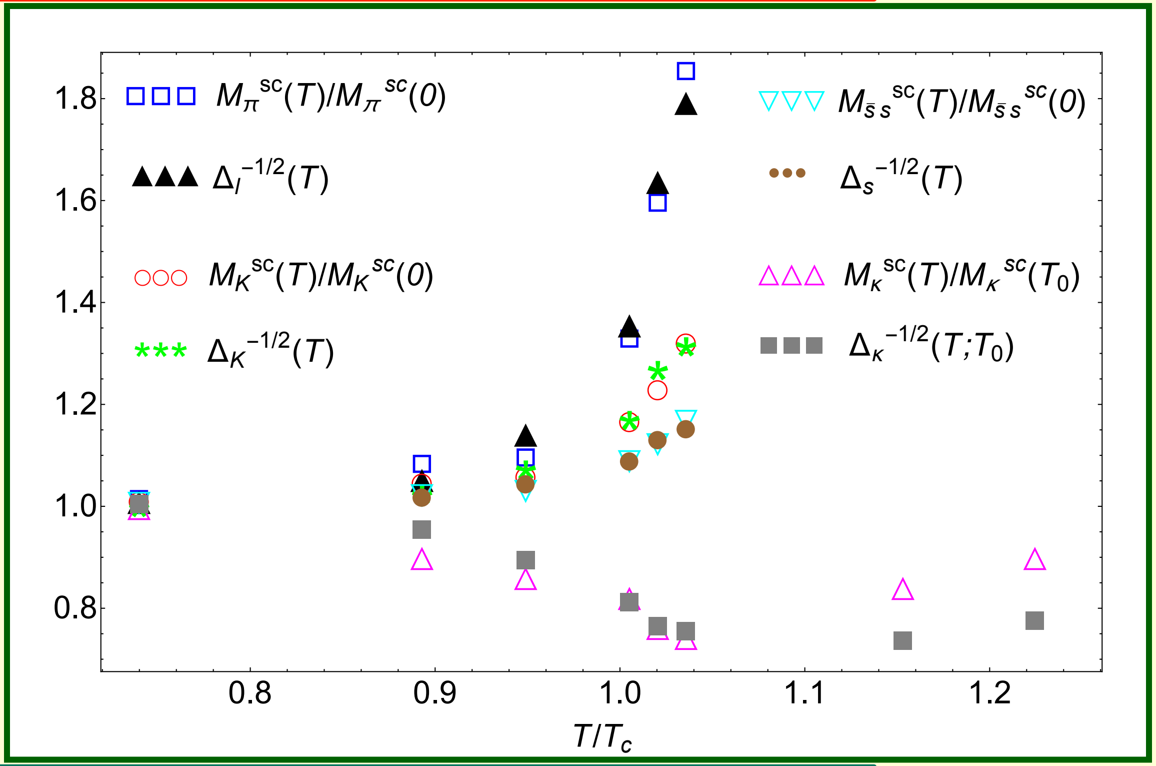
$$\frac{M_{\bar{s}s}^{sc}(T)}{M_{\bar{s}s}^{sc}(0)} \sim \left[\frac{\chi_P^{\bar{s}s}(0)}{\chi_P^{\bar{s}s}(T)}\right]^{1/2} \sim \left[\frac{\langle\bar{s}s\rangle(0)}{\langle\bar{s}s\rangle(T)}\right]^{1/2}$$

$$\frac{M_\kappa^{sc}(T)}{M_\kappa^{sc}(0)} \sim \left[\frac{\chi_S^\kappa(0)}{\chi_S^\kappa(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)-2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T)-2\langle\bar{s}s\rangle(T)}\right]^{1/2}$$

Anomalous contrib. $\frac{\hat{m}}{m_s}$ **suppressed**

WI AND LATTICE SCREENING MASSES

Same lattice setup for masses (Cheng et al EPJC'11) and condensates (PRD'08)



- $<$ **5% deviations** below T_c from predicted WI scaling
- Δ_i **subtracted condensates** with two fit parameters
- **Rapid** T_c **increase** in $M_\pi^{sc} \sim \langle \bar{q}q \rangle_l^{-1/2}$.
  Softer $M_K^{sc} \sim (\langle \bar{q}q \rangle_l + 2\langle \bar{s}s \rangle)^{-1/2}$. **Even softer** $M_{\bar{s}s}^{sc} \sim \langle \bar{s}s \rangle^{-1/2}$
- κ **minimum** from condensate diff. (last two points not fitted)

WI and Screening Masses

Subtracted Condensates have the right critical behavior in lattice, avoiding $T=0$ finite-size divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \ldots$:

$$\Delta_l(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + \langle \bar{q}q \rangle_l^{ref}}{\langle \bar{q}q \rangle_l^{ref}}$$

$$\Delta_K(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}$$

$$\Delta_s(T) = \frac{2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{s}s \rangle^{ref}}{\langle \bar{s}s \rangle^{ref}}$$

$$\Delta_\kappa(T;T_0) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l (T_0) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T_0) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}$$

$$\begin{aligned} r_1^3 \langle \bar{q}q \rangle_l^{ref} &= 0.750 \\ r_1^3 \langle \bar{s}s \rangle^{ref} &= 1.061 \\ r_1 &\simeq 0.31 \text{ fm} \end{aligned}$$

Screening vs pole masses

Lattice parametrization for inverse pseudo correlator (Karsch et al 2003):

$$K_P^{-1}(\omega,\vec{p}) \sim -\omega^2 + A^2(T)|\vec{p}|^2 + M^{pole}(T)^2$$

$$A(T) = \frac{M^{pole}(T)}{M^{sc}(T)}$$

Pseudoscalar susceptibility: $\chi_P = \dfrac{N_\chi}{M^2 + \Sigma_T(0,0)}$

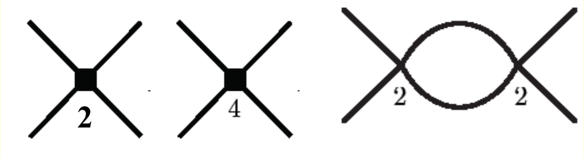
$p = 0$ **expansion:** $\Sigma(\omega,\vec{p};T) = \Sigma_T(0,0) + \alpha(T)\omega^2 - \beta(T)|\vec{p}|^2 + \mathcal{O}(p^4)$

$$\Rightarrow \quad A^2(T) = \frac{1+\beta(T)}{1+\alpha(T)}$$

$$\left[M^{pole}(T)\right]^2 = \frac{M^2 + \Sigma_T(0,0)}{1+\alpha(T)}$$

Therefore, $N_\chi \chi_P^{-1}(T) = [1+\alpha(T)]\, A^2(T)\, [M^{sc}(T)]^2$

Unitarizing scattering: resonances



ChPT Partial waves $\boxed{t^{IJ} = t_2^{IJ} + t_4^{IJ} + \ldots}$

Unitarity $\rightarrow$ **Im** $t(s) = \sigma(s)|t(s)|^2 \;\; (s \geq 4M^2) \Rightarrow$ $\boxed{\textbf{Im}\; t^{-1} = -\sigma}$

$\sigma(s) = \sqrt{1 - \frac{4M^2}{s}} = \frac{p_{CM}}{\sqrt{s}}$ **two-particle phase space**

$$\boxed{t^U(s;T) = \frac{\left[t_2(s)\right]^2}{t_2(s) - t_4(s;T)}}$$

(IAM)

Exactly proven for large NGB and chiral limits:
S.Cortés, AGN, J.Morales '16

FINITE TEMPERATURE:

$$t_4(s) \rightarrow t_4(s;T)$$

$$\sigma \rightarrow \sigma\left[1 + 2n_B(\sqrt{s}/2)\right] \equiv \sigma_T$$

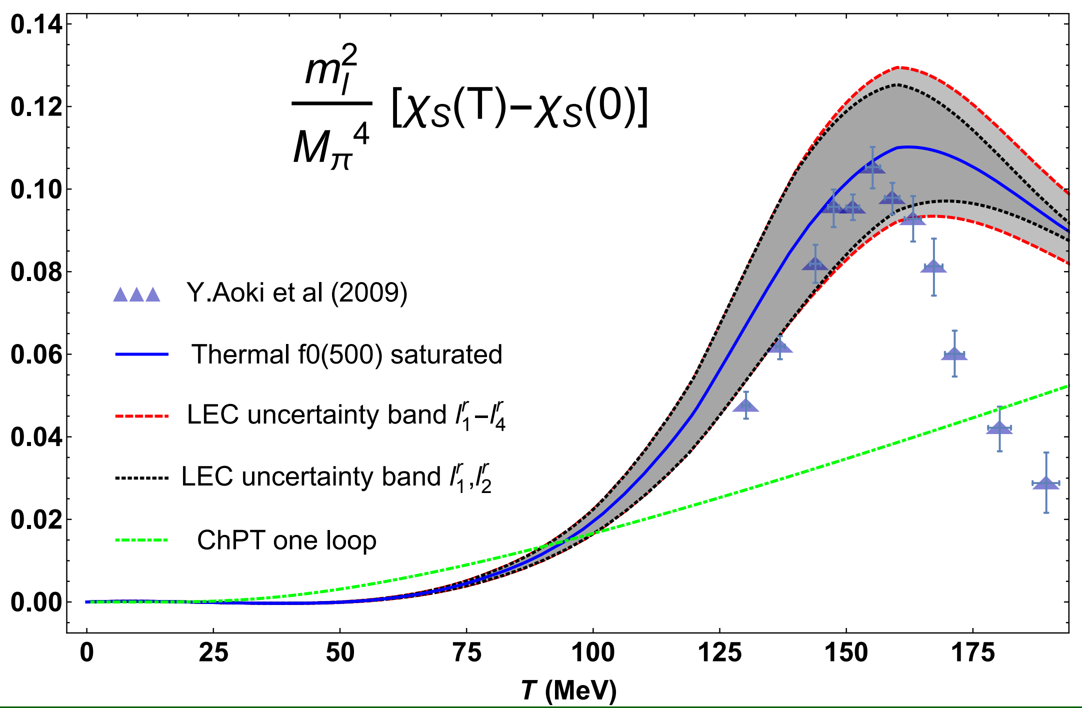
Thermal phase Space.
Bose net enhancement $(1+n)^2 - n^2$

A.Dobado, D.Fernández-Fraile, AGN, F.J.Llanes-Estrada, J.R.Peláez,E.Tomás-Herruzo, '02 '05 '07

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, 2018

$$\chi_S^U(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}$$



- Consistent with lattice transition peak.
- LECs and uncertaintities from unitarized $T=0$ fit in Hanhart, Peláez, Ríos PRL100 (2008)

$$s_p = 446.5 - i220.4 \text{ MeV}$$

- Consistent T_c reduction and χ_S growth near chiral limit

Saturated Scalar susceptibility in the LSM

S.Ferreres, AGN, A.Vioque, 2018

$$\mathcal{L}_{LSM} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \frac{\lambda}{4}\left[\Phi^T\Phi - v_0^2\right]^2 + h\sigma,$$

$$\chi_S(T) \;=\; \left(\frac{d^2h}{dm_q^2}\right) v(T) + \left(\frac{dh}{dm_q}\right)^2 \Delta_\sigma(k=0;T),$$

suppressed near T_c

$$M_{0\pi}^2 = \frac{h}{v} = \lambda(v^2 - v_0^2) \quad , \quad M_{0\sigma}^2 = M_{0\pi}^2 + 2\lambda v^2, \quad v = \langle\sigma\rangle$$

calculating the self-energy Σ to one loop:

M_π (MeV)	M_p (MeV)	Γ_p (MeV)	λ
0	450.0	172.5	8.4
0	775.1	550.0	20.0
140	450.0	159.2	9.6
140	750.1	550.0	21.2

$$\frac{\chi_S(T)}{\chi_S(0)} \simeq \frac{M_{0\sigma}^2 + \Sigma\,(k=0;T=0)}{M_{0\sigma}^2 + \Sigma\,(k=0;T)}$$