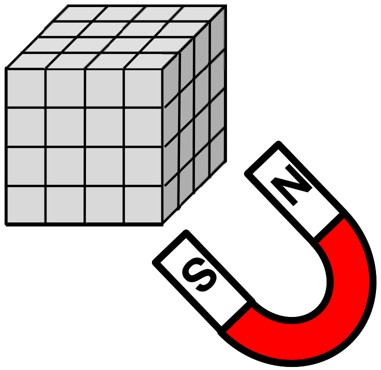
Rotation on lattices

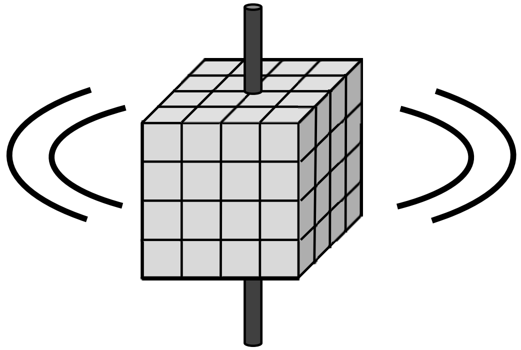
Arata Yamamoto (University of Tokyo)

Magnetism & Rotation

magnetic field



rotation

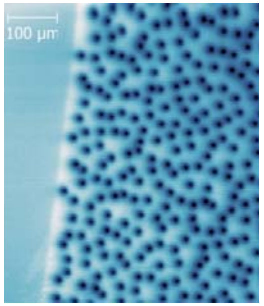


similar, but different

Magnetism & Rotation

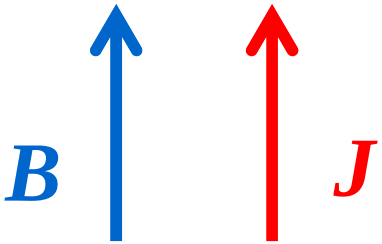
magnetic field

vortex in superconductors



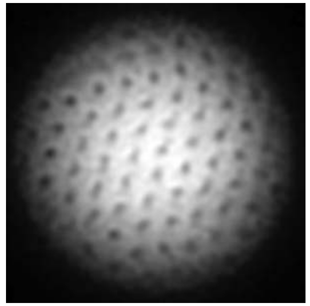
Wells, Pan, Wang, Fedoseev, Hilgenkamp (2015)

chiral magnetic effect



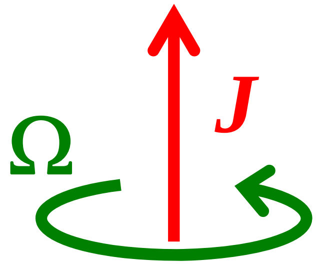
rotation

vortex in superfluids

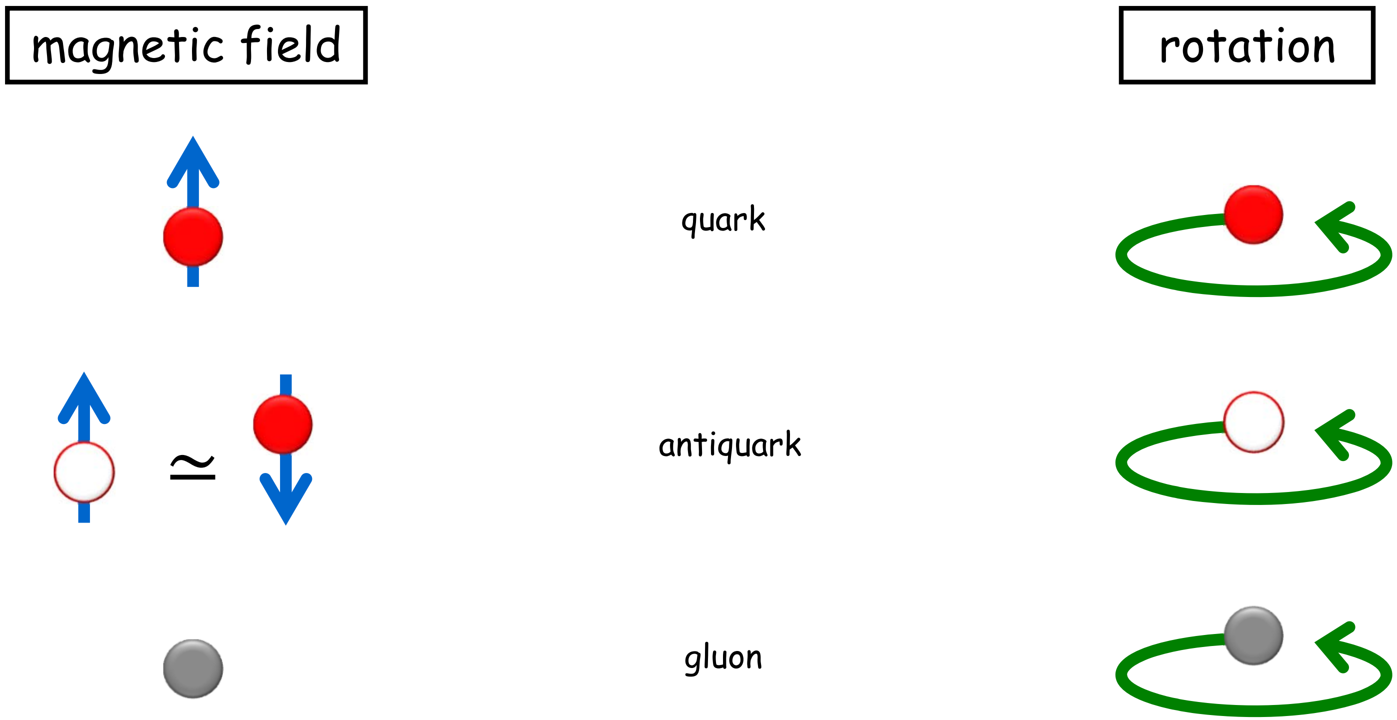


Zwierlein, Abo-Shaeer, Schirotzek, Schunck, Ketterle (2005)

chiral vortical effect



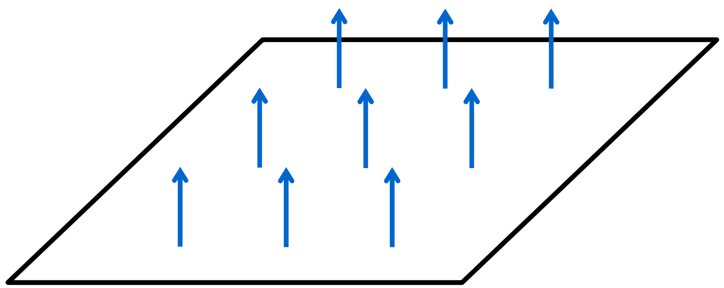
Magnetism & Rotation



$S_G + S_F(B)$ $\quad$ $S_G(\Omega) + S_F(\Omega)$

Magnetism & Rotation

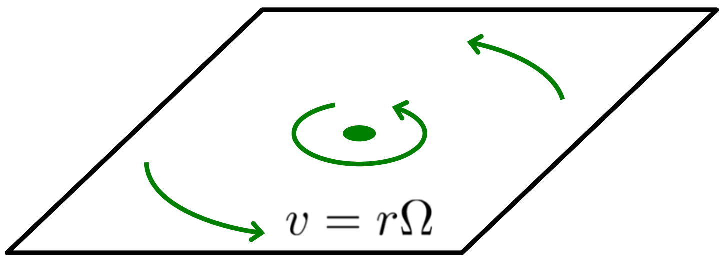
magnetic field



homogeneous

infinite volume

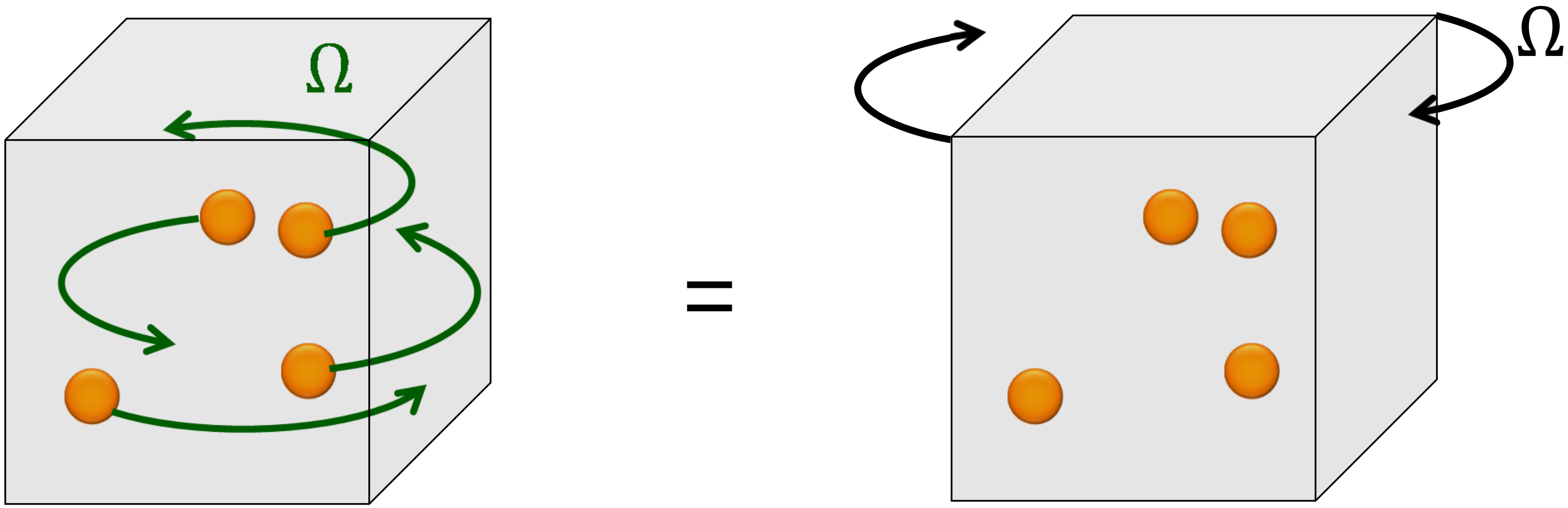
rotation



inhomogeneous

boundary s.t. $v < c$

Simulation of rotation



rest frame

rotating frame

Simulation of rotation

1. non-relativistic rotation $r\Omega \ll c$

   rotating atomic & molecular gases, etc.

2. relativistic rotation $r\Omega \sim c$

   rotating QGP & neutron stars

Non-relativistic rotation

in non-relativistic theories,

$$S(\Omega) = S_0 - \Omega L_z$$

Non-relativistic rotation

non-relativistic Bose gas

$$S(\Omega) = S_0 - \Omega L_z$$

$$S_0 = \int d\tau d^3x \;\; \Phi^* \left(\frac{\partial}{\partial \tau} - \mu - \frac{1}{2m} \Delta \right) \Phi + V$$

$$L_z = -i \int d\tau d^3x \;\; \Phi^* \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \Phi$$

Non-relativistic rotation

non-relativistic Bose gas

$$S(\Omega) = S_0 - \Omega L_z$$

$$S_0 = \int d\tau d^3x \;\; \Phi^* \left(\frac{\partial}{\partial \tau} - \mu - \frac{1}{2m}\Delta \right) \Phi + V$$

anti-Hermitian ⟶ complex Langevin simulation

Hayata, Yamamoto (2015)

$$L_z = -i \int d\tau d^3x \;\; \Phi^* \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \Phi$$

Relativistic rotation

in relativistic theories,

$$\text{rotating frame} \; = \; \text{curved space} \quad g_{\mu\nu}(x)$$

Relativistic rotation

Dirac fermion in a rotating frame

$$S_F(\Omega) = \int d^4x\, \bar{\psi} \left[m + \gamma^1 D_x + \gamma^2 D_y + \gamma^3 D_z + \gamma^4 D_\tau \right.$$

$$\left. - i\Omega\gamma^4 \left(x D_y - y D_x \right) - \frac{\Omega}{2} \gamma^4\, \sigma^{12} \right] \psi$$

↑ orbit-rotation coupling

↑ spin-rotation coupling

Relativistic rotation

Wilson fermion in a rotating frame   Yamamoto, Hirono (2013)

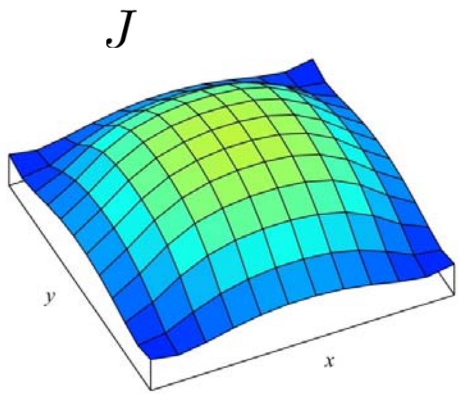
$$S_F(\Omega) = \sum_x \bar{\psi}\left[1 - \kappa\{(1-\gamma^1 - i\Omega y\gamma^4)T_{x+} + (1+\gamma^1 + i\Omega y\gamma^4)T_{x-}\right.$$

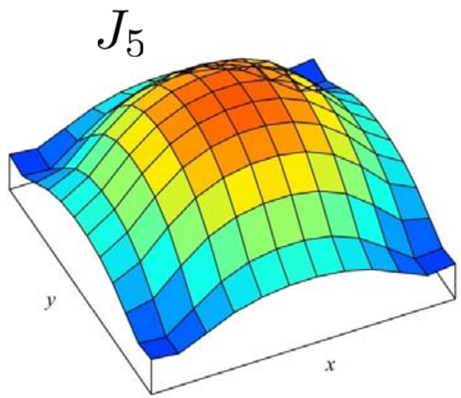
$$+\cdots$$

$$\left. +(1-\gamma^4)e^{-\frac{\Omega}{2}\sigma^{12}}T_{\tau+} + (1+\gamma^4)e^{\frac{\Omega}{2}\sigma^{12}}T_{\tau-}\}\right]\psi$$

Relativistic rotation

chiral vortical effect (free fermion)





analytical formula

$$J = \frac{\mu\mu_5}{\pi^2}\Omega$$

$$J_5 = \left(\frac{\mu^2 + \mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\Omega$$

Relativistic rotation

open problems in interacting theories:

1. sign problem

2. inhomogeneous renormalization

Open problem 1: sign problem

Wilson fermion in a rotating frame

$$S_F(\Omega) = \sum_x \bar{\psi}\,[1 - \kappa\{(1 - \gamma^1 - i\Omega y\gamma^4)T_{x+} + (1 + \gamma^1 + i\Omega y\gamma^4)T_{x-}$$

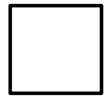
$$+ \cdots$$

$$+(1 - \gamma^4)e^{-\frac{\Omega}{2}\sigma^{12}}T_{\tau+} + (1 + \gamma^4)e^{\frac{\Omega}{2}\sigma^{12}}T_{\tau-}\}]\psi$$

not γ_5-Hermitian = complex

Open problem 1: sign problem

gluon in a rotating frame

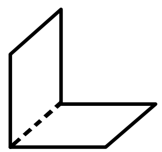
discretized as “plaquette” 

$$S_G(\Omega) = \int d^4x \, \frac{1}{g^2} \, \mathrm{tr} \Big[F_{x\tau}F_{x\tau} + F_{y\tau}F_{y\tau} + F_{z\tau}F_{z\tau}$$

$$+ (1 - r^2\Omega^2)F_{xy}F_{xy} + (1 - y^2\Omega^2)F_{xz}F_{xz} + (1 - x^2\Omega^2)F_{yz}F_{yz}$$

$$- i2y\Omega F_{xy}F_{y\tau} + i2x\Omega F_{yx}F_{x\tau} - i2y\Omega F_{xz}F_{z\tau} + i2x\Omega F_{yz}F_{z\tau}$$

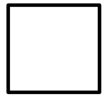
$$- 2xy\Omega^2 F_{xz}F_{zy} \Big]$$

discretized as “chair-type” loop 

Open problem 1: sign problem

gluon in a rotating frame

discretized as "plaquette"



$$S_G(\Omega) = \int d^4x \, \frac{1}{g^2} \, \mathrm{tr} \Big[F_{x\tau}F_{x\tau} + F_{y\tau}F_{y\tau} + F_{z\tau}F_{z\tau}$$

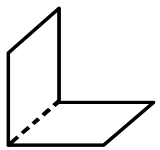
$$+(1 - r^2\Omega^2)F_{xy}F_{xy} + (1 - y^2\Omega^2)F_{xz}F_{xz} + (1 - x^2\Omega^2)F_{yz}F_{yz}$$

$$-i2y\Omega F_{xy}F_{y\tau} + i2x\Omega F_{yx}F_{x\tau} - i2y\Omega F_{xz}F_{z\tau} + i2x\Omega F_{yz}F_{z\tau}$$

complex

$$-2xy\Omega^2 F_{xz}F_{zy} \Big]$$

discretized as "chair-type" loop



Open problem 1: sign problem

rotating frame

$$S_G(\Omega) + S_F(\Omega)$$

complex

chemical potential

$$S_G + S_F(\mu)$$

complex

sign problem !!

Open problem 1: sign problem

rotating frame

$$S_G(\Omega) + S_F(\Omega) \longrightarrow S_G(i\Omega) + S_F(i\Omega)$$

complex → real

$$\Omega_E = i\Omega$$

"Euclidean rotation"

Yamamoto, Hirono (2013)

chemical potential

$$S_G + S_F(\mu) \longrightarrow S_G + S_F(i\mu)$$

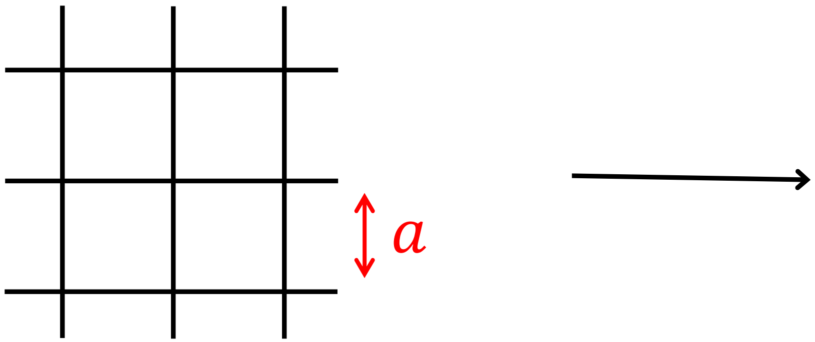
complex → real

$$\mu_I = i\mu$$

imaginary chemical potential

Open problem 2: inhomogeneous renormalization

flat lattice

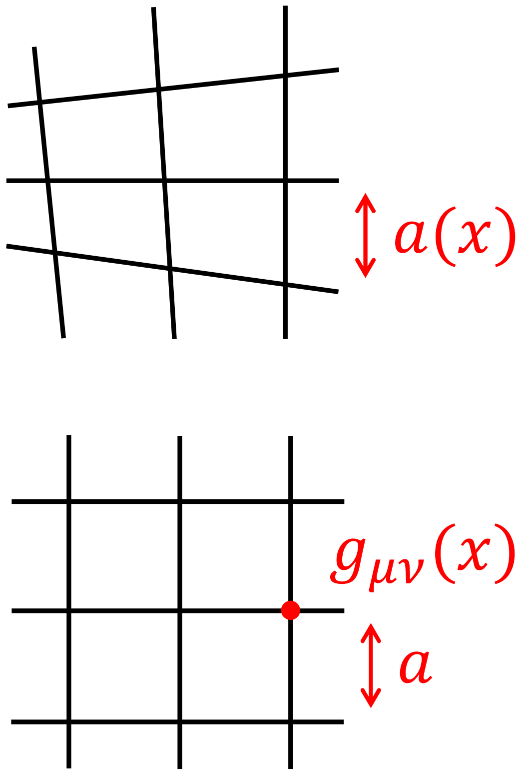


homogeneous renormalization

$$\langle \mathcal{O}_R(x) \rangle = C \langle \mathcal{O}(x) \rangle$$

Open problem 2: inhomogeneous renormalization

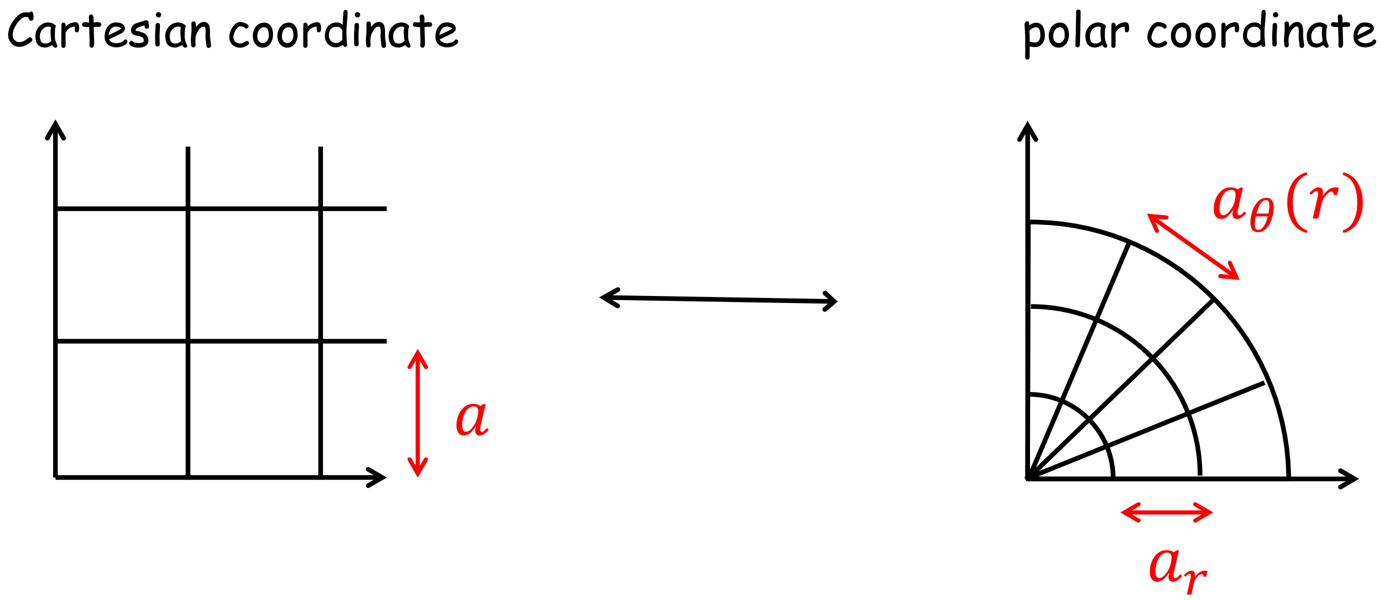
curved lattice



inhomogeneous renormalization

$$\langle \mathcal{O}_R(x) \rangle = C(x) \langle \mathcal{O}(x) \rangle$$

Open problem 2: inhomogeneous renormalization

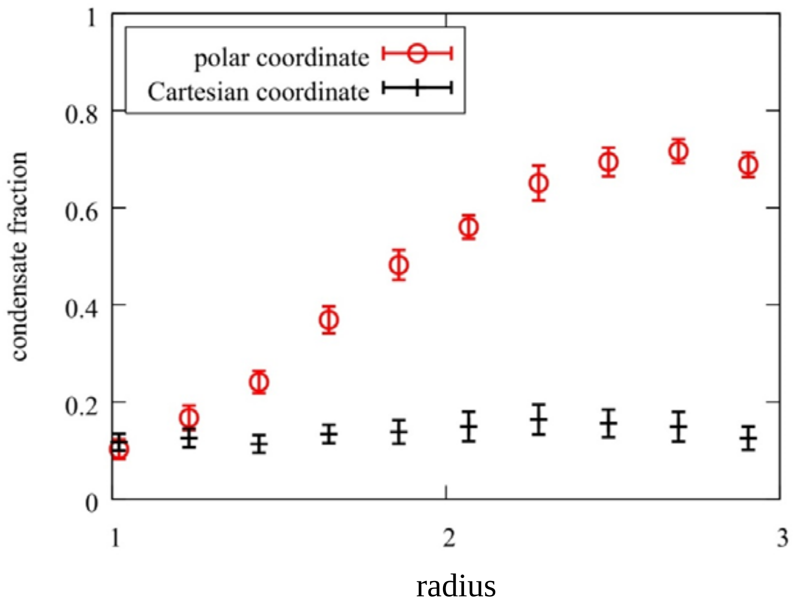


Open problem 2: inhomogeneous renormalization

lattice ϕ^4 theory

Benić, Yamamoto (2016)

$$\text{condensate fraction} = \frac{\langle \phi \rangle^2}{\langle \phi^2 \rangle}$$



Summary

✓ simulation of non-relativistic rotation is easy

✓ simulation of relativistic rotation is very challenging

sign problem ⟶ Euclidean rotation? model study?

inhomogeneous renormalization ⟶ Ω-expansion?