Exploring the existence of inhomogeneous phases in the 2+1-dimensional Gross-Neveu model in the limit of infinitely many flavors (part 1 – continuum approach)

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CRC-TR 211 Strong-interaction matter under extreme conditions

Motivation

- Phase diagram of QCD ... to difficult
- Consider simpler model $\longrightarrow$ Gross-Neveu model
- QFT easier for smaller spacetime dimension
  - $d = 1 + 1$ O. Schnetz, M. Thies and K. Urlichs, Annals Phys. 314, 425 (2004)
  - $d = 1 + 2$ K. Klimenko, Z. Phys. C 37, 457 (1988)
- Are there inhomogeneous phases in $d = 2 + 1$?
  - "yes" M. Winstel, J. Stoll and M. Wagner, arXiv:1909.00064 [hep-lat]
  - "no" R. Narayanan, Phys. Rev. D 101, no.9, 096001 (2020)

Question

Under which conditions is an inhomogeneous phase favored?

Techniques

- Stability analysis (part 1)
- Global minimization on lattice (part 2)

Gross-Neveu model in 2+1 dimensions

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/)\psi - \frac{\lambda}{2N_f}(\bar{\psi}\psi)^2$$

- N_f quarks $\psi = (\psi_1, \ldots, \psi_{N_f})$
- $[\lambda] = -1$
- Non-perturbatively renormalizable

  B. Rosenstein, B. J. Warr and S. H. Park, Phys. Rev. D 39, 3088 (1989)

Fermions in 2+1 dimensions

- Dirac algebra has 2 different irreps of dim 2
- Use direct sum of these to talk about (discrete) chiral symmetry
- $N_d = 2$ or 4 does not change the results

Mean field approximation

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(\int_0^\beta \mathrm{d}\tau \int_V \mathrm{d}^2 x \left[\bar{\psi}(\mathrm{i}\partial\!\!\!/)\psi - \frac{N_f}{2\lambda}(\bar{\psi}\psi)^2 + \mu\bar{\psi}\gamma^0\psi \right] \right)$$

Bosonize and integrate out the quarks $\downarrow$

$$\mathcal{Z} = \int \mathcal{D}\sigma \exp(-S_{\mathrm{eff}})$$

with

$$S_{\mathrm{eff}}[\sigma] = N_f \left(\frac{\beta}{2\lambda} \int_V \mathrm{d}^2 x\, \sigma^2 - \log\left[\det_{n,\vec{p},\mathrm{d}} \left(\mathrm{i}\partial\!\!\!/ + \mu\gamma^0 + \sigma \right) \right] \right)$$

Mean field approximation $\longrightarrow$ minimize S_{eff} w.r.t. σ

Regularization

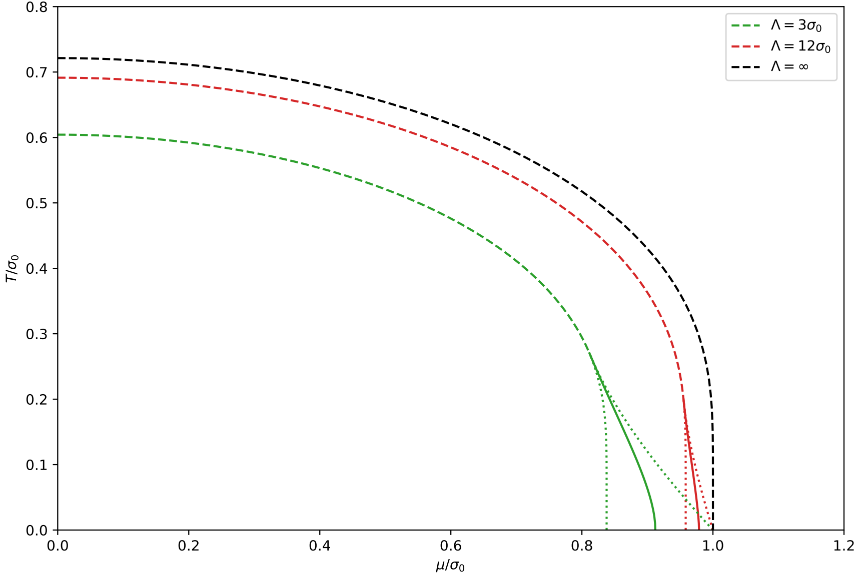
- The effective action diverges
- Use Pauli–Villars scheme for regularization

$$\int_0^\infty \mathrm{d}p\, f(M^2) \to \int_0^\infty \mathrm{d}p \left[f(M^2) - f(M^2 + \Lambda^2) \right]$$

- Lorentz invariant
- Apply only to vacuum part
- Write everything in units of $\sigma_0 =$VEV

$$\lambda = \frac{4\pi}{N_d \left(\sqrt{\sigma_0^2 + \Lambda^2} - |\sigma_0| \right)}$$

Homogeneous phase diagram



Stability analysis

$$\log \det_{n,\vec{p},\mathrm{d}} \left(\mathrm{i}\partial\!\!\!/ + \mu\gamma^0 + \sigma\right) = \mathrm{Tr}\log\left(\mathrm{i}\partial\!\!\!/ + \mu\gamma^0 + \sigma\right)$$

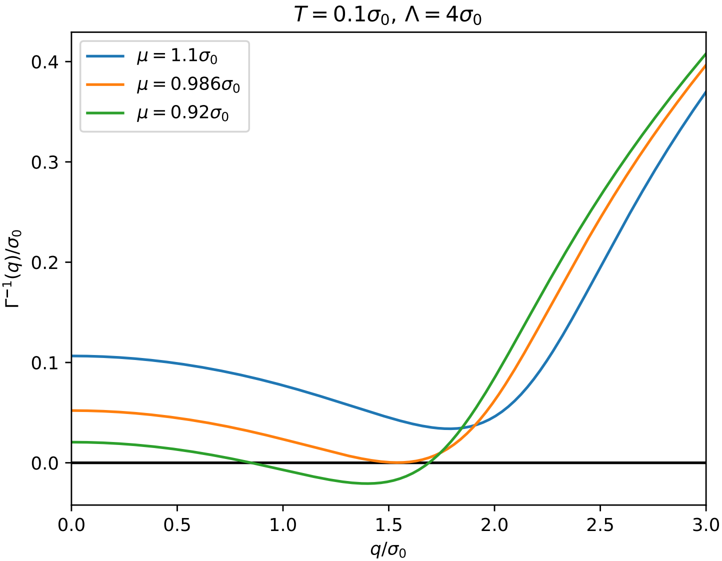
- Inversion of propagator only possible for constant mass
  $\longrightarrow$ Expand around homogeneous field configuration
- Use global homogeneous minimum as expansion point

$$\left.\frac{\delta\Omega}{\delta\sigma(x)}\right|_{\text{homogeneous minimum}} = 0$$

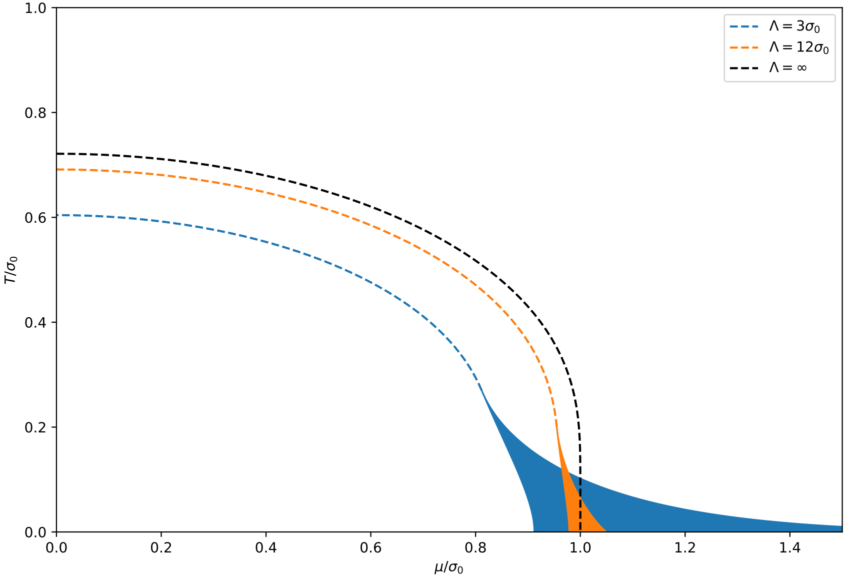
- Stability depends on second derivative

$$\left.\frac{\delta^2\Omega}{\delta\sigma(x)\delta\sigma(y)}\right|_{\text{homogeneous minimum}} \xrightarrow{\text{diagonalize}} \Gamma^{-1}(q)$$

Stability analysis



Phase diagram



Analytical results

- Renormalized homogeneous condensate

$$\sigma(\mu, T) = \begin{cases} T \operatorname{arcosh}\left(\frac{1}{2}e^{\sigma_0/T} - \cosh(\mu/T)\right) & \text{if } T < \frac{\sigma_0}{\log(4)} \text{ and } \mu < \mu_{\text{boundary}}(T) \\ 0 & \text{else} \end{cases}$$

where $\mu_{\text{boundary}}(T) = T \operatorname{arcosh}\left(\frac{1}{2}e^{\sigma_0/T} - 1\right)$

- Second order Phase boundary at finite regulator

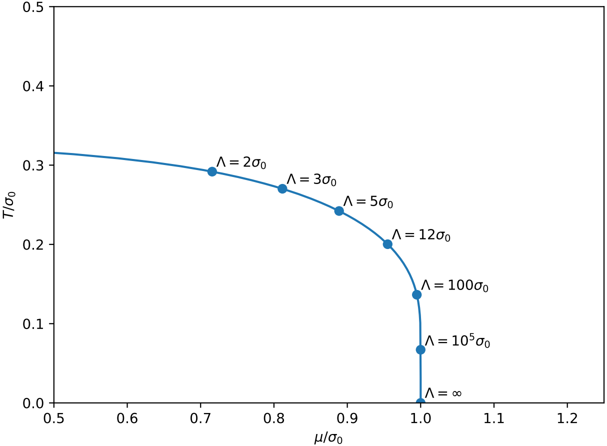
$$\mu(T) = T \operatorname{arcosh}\left(\frac{1}{2}e^{s/T} - 1\right), \qquad T < \frac{s}{\log(4)}$$

where $s = |\sigma_0| + \Lambda - \sqrt{\sigma_0^2 + \Lambda^2}$

- Lifshitz point = tricritical point

$$T_{LP}(\Lambda) = \frac{-s}{W_{-1}\left(-\frac{s}{2\Lambda}\right)} \qquad \mu_{LP}(\Lambda) = T_{LP} \operatorname{arcosh}\left(\frac{\Lambda}{T_{LP}} - 1\right)$$

Lifshitz point



Result (part 1)

Inhomogeneous phase disappears as $\Lambda \to \infty$