Meson condensation in chiral perturbation theory

Massimo Mannarelli
INFN-LNGS
massimo@lngs.infn.it

Hot Problems of Strong Interactions, Nov 12th, 2020

OUTLINE

- **Hadronic phases**
- **Symmetries and symmetry breaking**
- **Methods and Comparison**
- **Multimeson condensation**
- **Solitonic phase**

particles

MDPI

Review

Meson Condensation

Massimo Mannarelli

INFN-Laboratori Nazionali del Gran Sasso, Via G. Acitelli, 22, I-67100 Assergi (AQ), Italy;
massimo.mannarelli@lngs.infn.it

Received: date; Accepted: date; Published: date

check for updates

Abstract: We give a pedagogical review of the properties of the various meson condensation phases triggered by a large isospin or strangeness imbalance. We argue that these phases are extremely interesting and powerful playground for exploring the properties of hadronic matter. The reason is that they are realized in a regime in which various theoretical methods overlap with increasingly precise numerical lattice QCD simulations, providing insight on the properties of color confinement and of chiral symmetry breaking.

Keywords: quark matter; QCD phase diagram; nuclear matter

Particles 2 (2019) 3, 411-443 • e-Print: 1908.02042

open access

Hadronic phases

Standard hadronic matter is tough

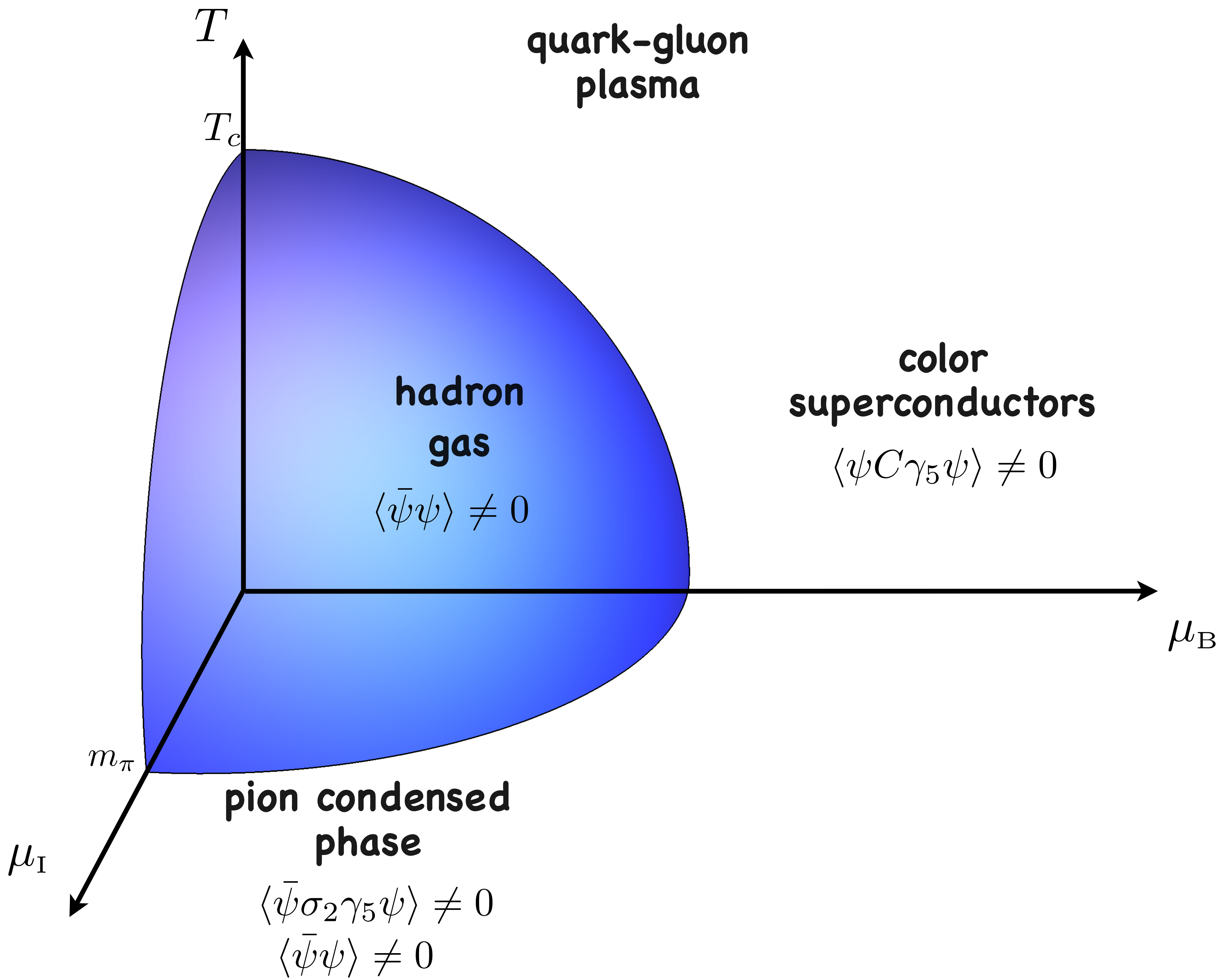
- We would like to pull the reductionist approach to the nucleon constituents

- Unfortunately hadronic matter has two peculiar properties that make our understanding difficult

Confinement

Non Perturbative Interactons

A view of the QCD phase diagram



Identify the QCD phases by their condensates

In each phase different quark condensates are realized

Hadron gas	Color superconductors
chiral condensate	diquark condensate
$\langle\bar{\psi}\psi\rangle$	$\langle\psi C\gamma_5\psi\rangle$

Meson superfluid	Quark-gluon plasma
pion condensate	no condensate
$\langle\bar{\psi}\sigma_2\gamma_5\psi\rangle$	

Each condensate **breaks** or **locks** the QCD symmetries

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$$

in a different way

How to make progress

Any **route** and any **physically sound method** should be explored

•Any route: for me means going beyond the physical sheet

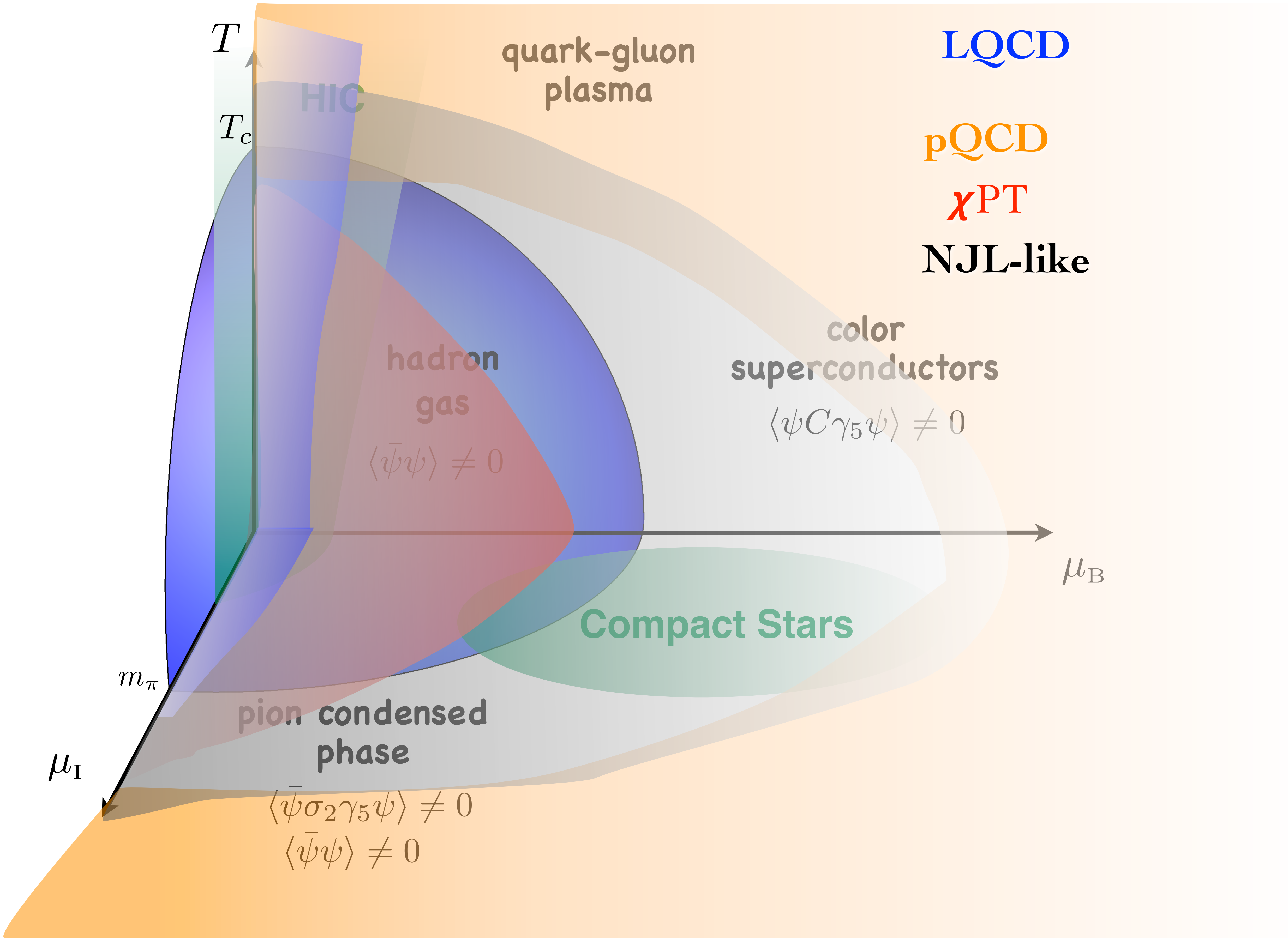
• Methods: NJL, ChiPT, Perturbation Theory, Sum Rules etc.

The QCD exploration is hard to realize in labs. Two main ways

Heavy ion collisions
(hot matter)

Compact stars
(dense catalyzed matter)

Theoretical/numerical/experimental tools



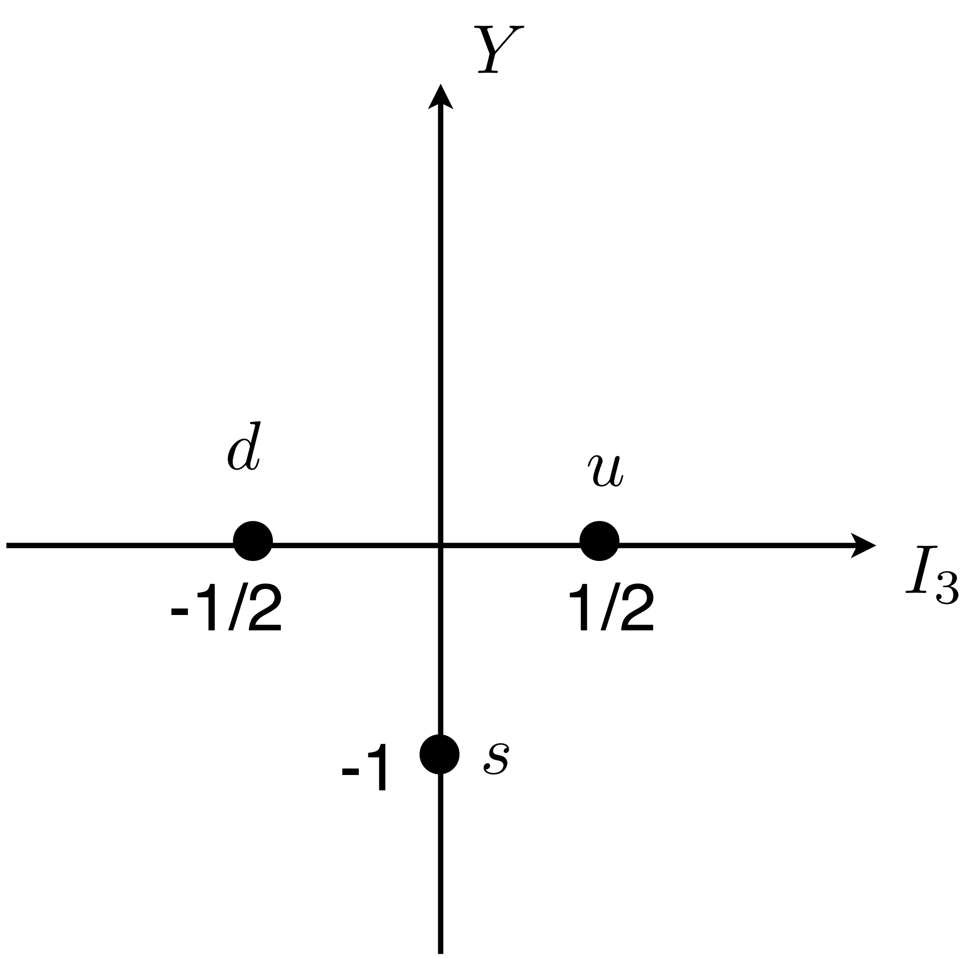
Symmetries and symmetry breaking

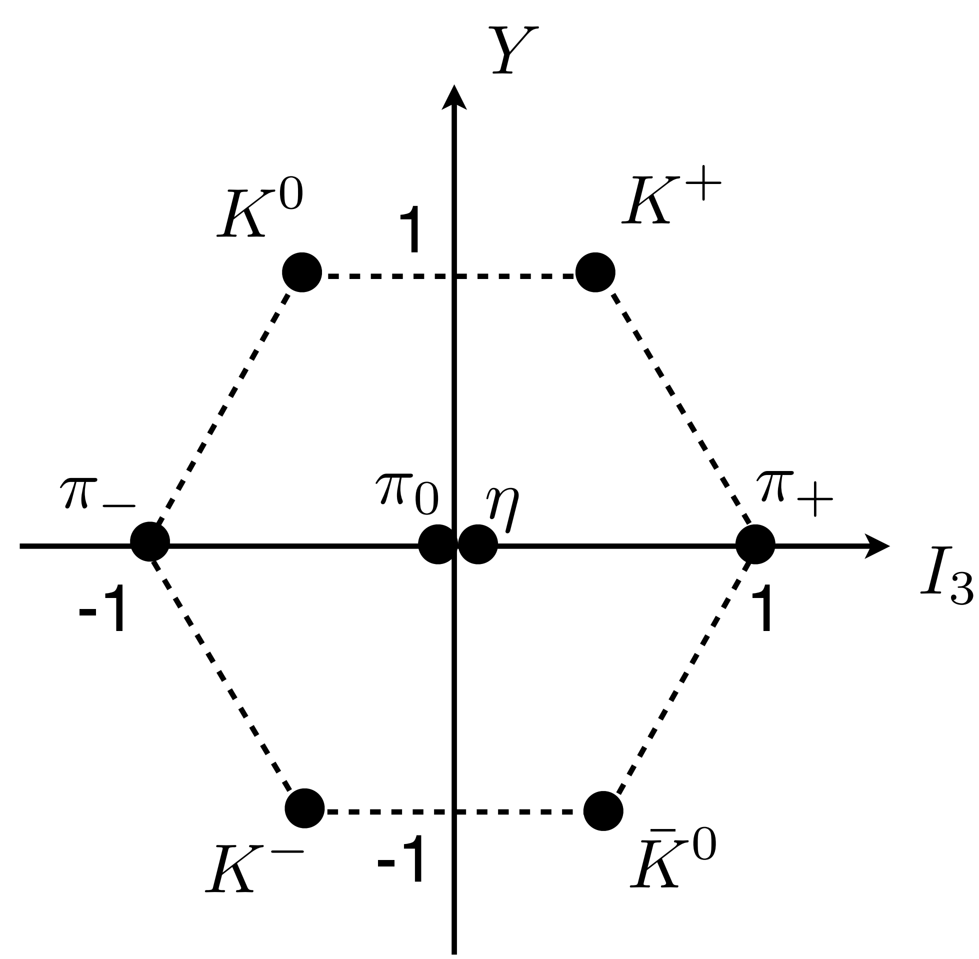
Classification of mesons

Global symmetries of QCD

$$\mathrm{G} = \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B$$

Ypercharge-Isospin diagram





Effect of isospin

pions

Energy spectrum splitting
Stark-like effect

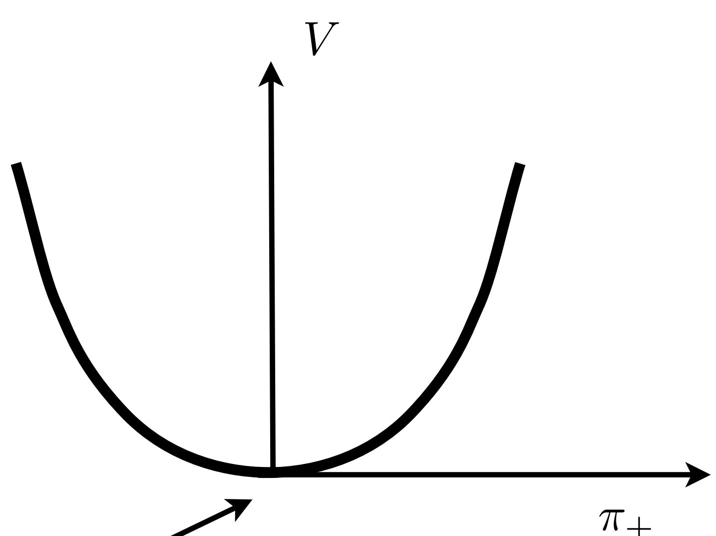
$$E_{\pi^0} = \sqrt{m_\pi^2 + p^2}$$
$$E_{\pi^-} = +\mu_I + \sqrt{m_\pi^2 + p^2}$$
$$E_{\pi^+} = -\mu_I + \sqrt{m_\pi^2 + p^2}$$

$$\boxed{m_{\pi^+}^{\text{eff}} = m_\pi - \mu_I}$$

What happens for $\mu_I > m_\pi$?

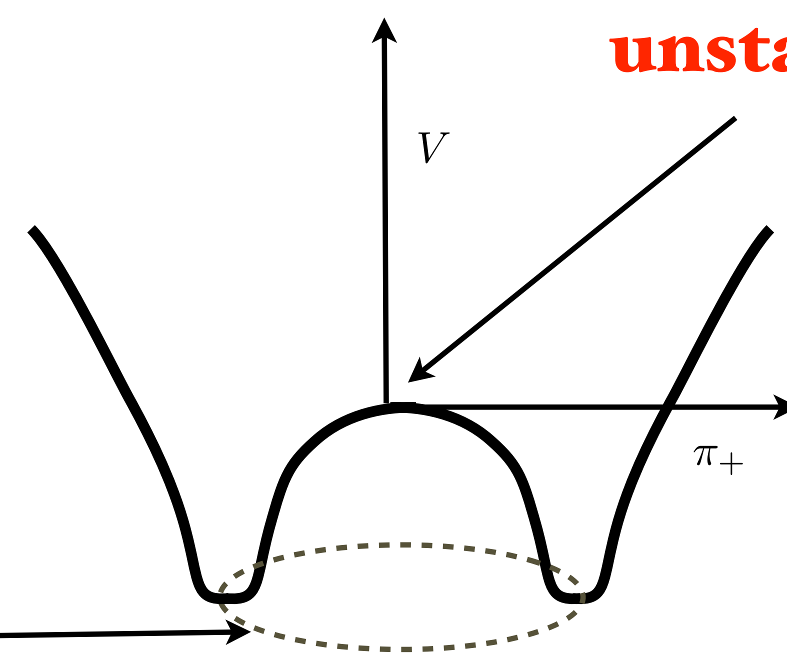
A vanishing "effective mass" may imply the onset of an instability because $(m_\pi^{\text{eff}})^2 \sim \dfrac{\partial^2 V}{\partial \pi^2}$

$\mu_I < m_\pi$



stable vacuum

$\mu_I > m_\pi$



unstable

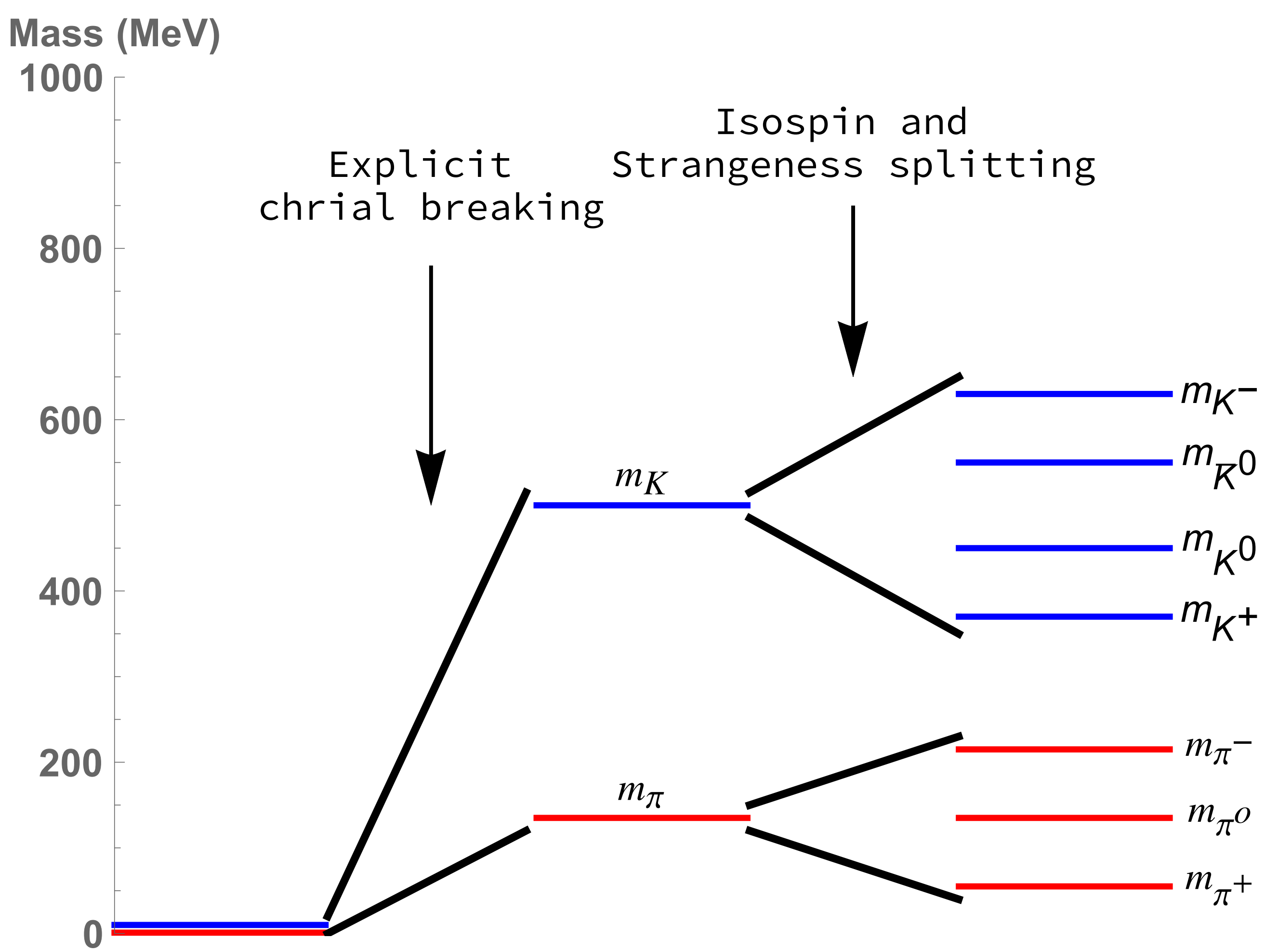
stable vacuum

Isospin and strangeness

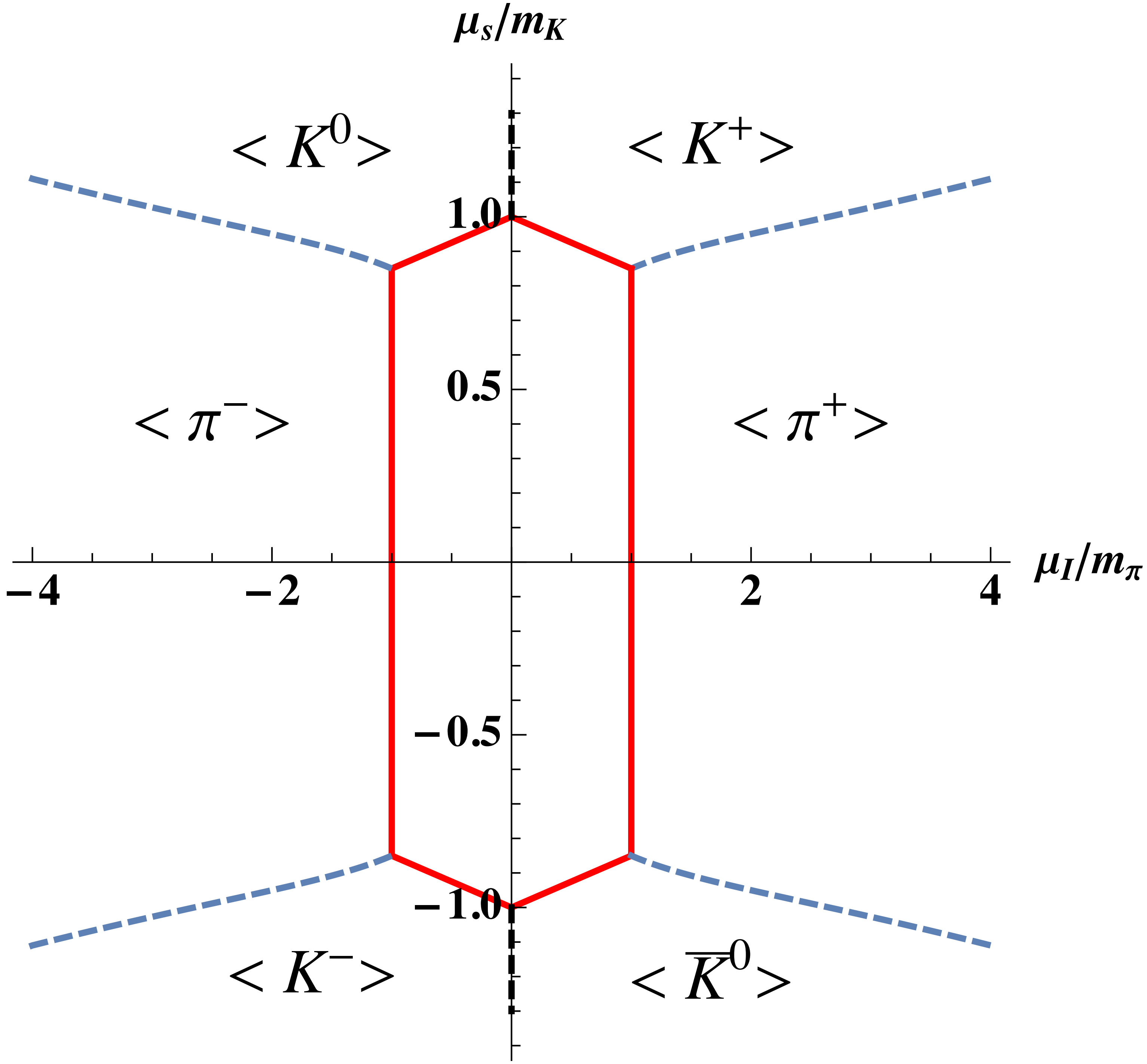
Mesons mass splitting

$$m_{\pi^0} = m_\pi$$
$$m_{\pi^\pm} = m_\pi \mp \mu_I$$

$$m_\eta = \sqrt{\frac{4m_K^2 - m_\pi^2}{3}}\,,$$
$$m_{K^\pm} = m_K \mp \frac{1}{2}\mu_I \mp \mu_S\,,$$
$$m_{K^0/\bar{K}^0} = m_K \pm \frac{1}{2}\mu_I \mp \mu_S\,,$$



Combining symmetries and mass splittings



solid line: second order
dotted line: first order

Kogut and Toublan PhysRevD.64.034007 (2001)
M.M. Particles 2 (2019) no.3, 411-443

The first order transitions indicate the possible coexistence of two mesonic condensates

Symmetry breaking path

massless quarks

$\psi_L \to U_L \psi_L$

$\psi_R \to U_R \psi_R$

$$\underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B$$

$\langle \bar{\psi}\psi \rangle \neq 0$

Spontaneous chiral symmetry breaking

invariant under locked chiral rotations

$U_L = U_R$

$$\underbrace{SU(2)_I \times U(1)_Y}_{\supset U(1)_Q} \times U(1)_B$$

Meson octet (Pseudo) Nambu-Goldstone bosons (massive quarks)

$\mu_I \neq 0$

$\mu_s \neq 0$

Explicit symmetry breaking

“normal phase” symmetry

$$\underbrace{U(1)_I \times U(1)_Y}_{\supset U(1)_Q} \times U(1)_B$$

Meson octet (no mass degeneracy)

Spontaneous phase locking

$\mu_I > m_\pi$

or

$\mu_s > m_k - \dfrac{m_\pi}{2}$

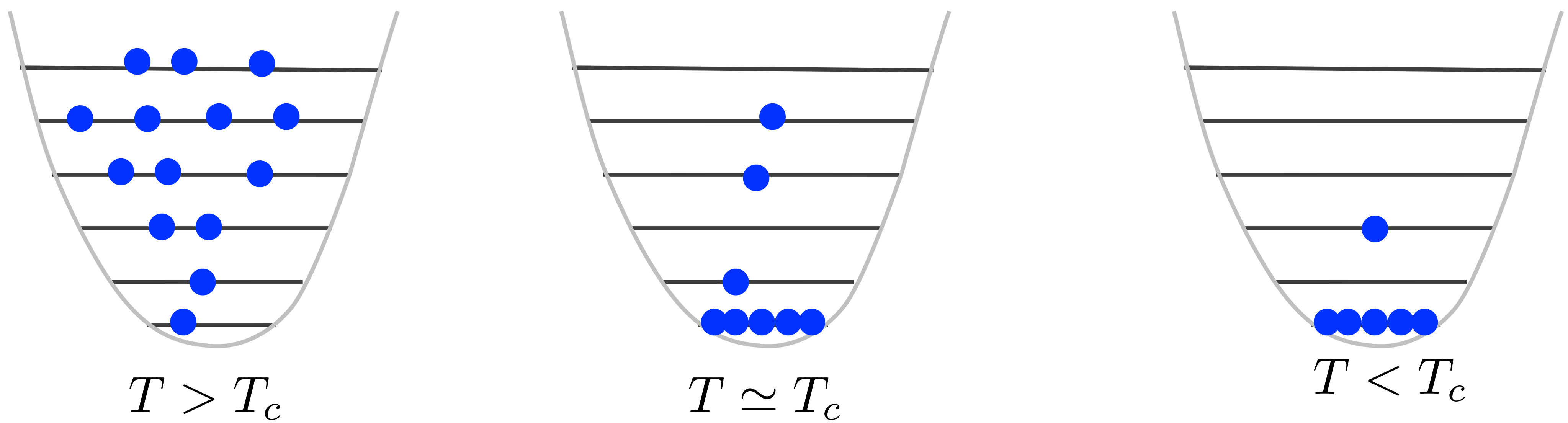
$$\underbrace{U(1) \times U(1)_B}_{\not\supset U(1)_Q}$$

One NGB

Bose-Einstein condensation

The Bose-Einstein condensate (BEC) is a **coherent state of matter.**
A "thermodynamically" large number of particles occupy the same quantum state

BOSONS@ low temperature in an harmonic potential



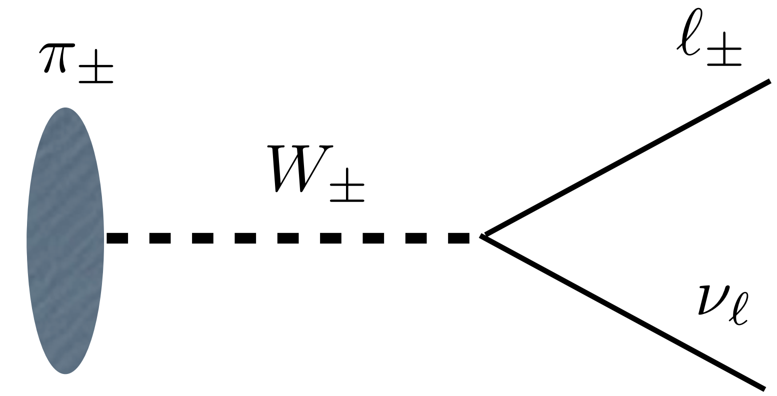
Requirements:

1. Particles must be **bosons** or boson-like, e.g. Cooper pairs in BCS
2. **Cold system**: A fight between thermal disorder and quantum coherence
3. Particles must be **stable**

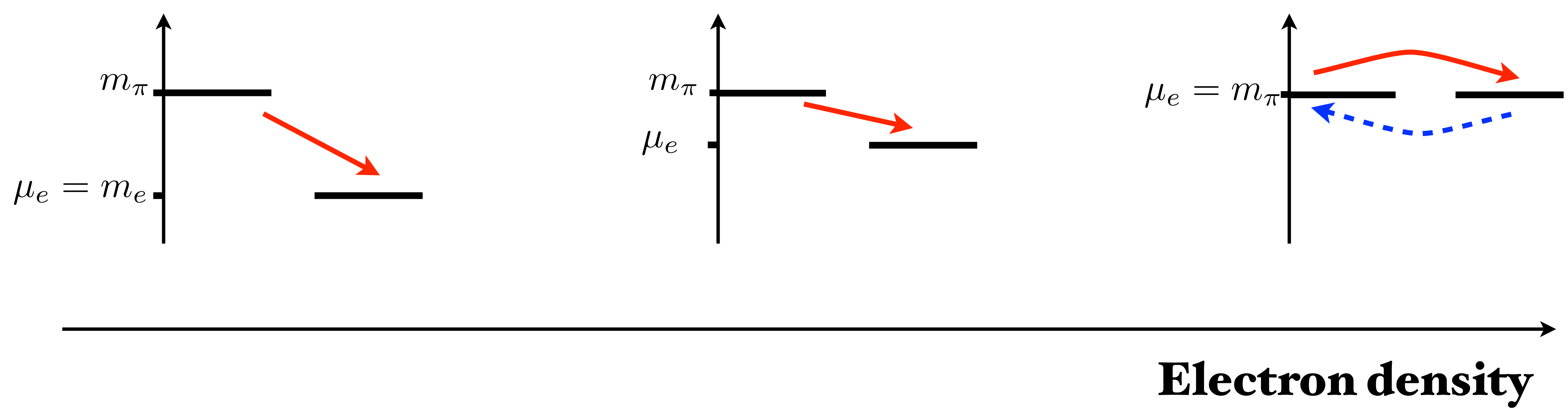
Condensate of mesons?

1. Mesons are **bosons**
2. Can be produced at **low temperature** e.g. inside compact stars
3. Mesons are <u>not</u> stable

pion decay



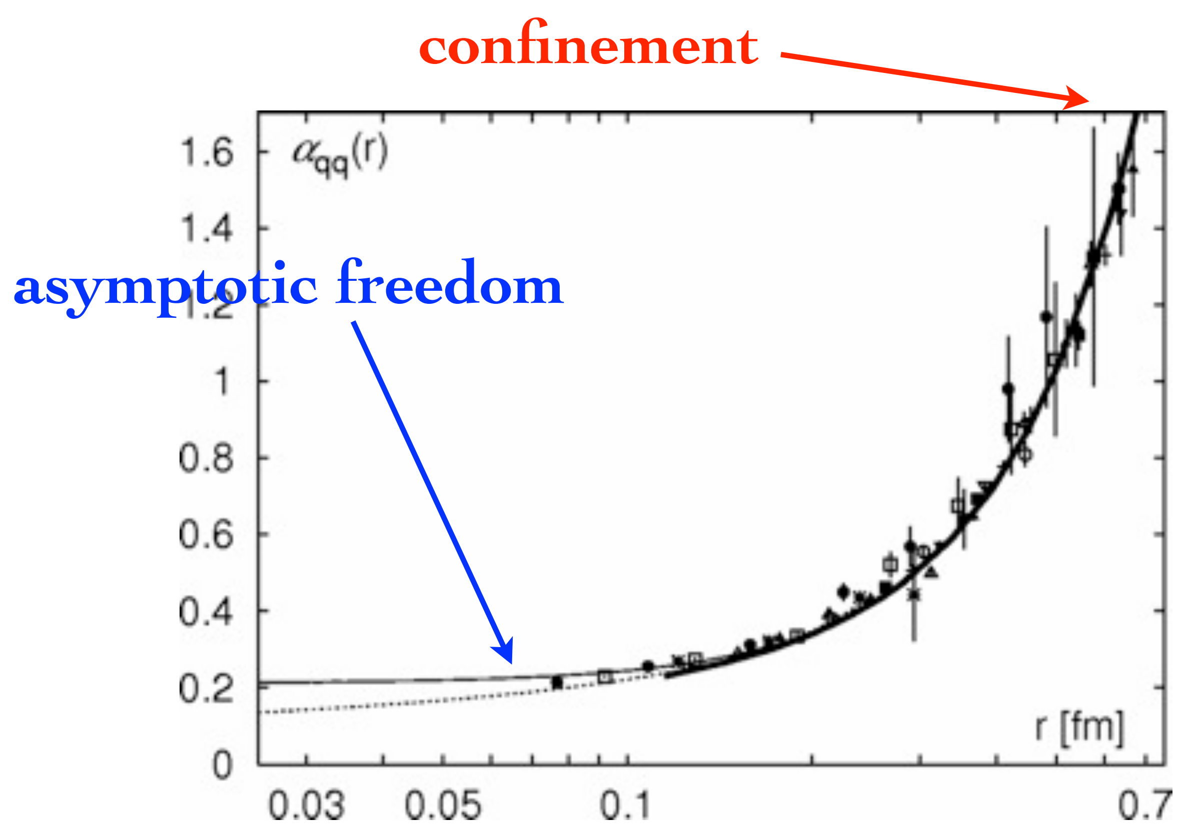
The pion decay is Pauli blocked if $\mu_\ell > m_\pi$ and the pion becomes stable



Methods

Lattice QCD

Compute nonperturbative quantities using the correct degrees of freedom



"Running" of the interaction strength.
LQCD perfectly reproduces the experimentally observed behavior

Kaczmarek and Zantow
Physical Review D 71(11):114510

LQCD cannot be (easily) applied for nonvanishing baryon and strangeness density

$$\mu_B \neq 0$$
$$\mu_S \neq 0$$

Still possible to use LQCD when

$$\mu_B = 0$$
$$\mu_S = 0$$
$$\mu_I \neq 0$$

NJL modeling

$$\mathcal{L}_{\rm NJL} = \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} + \mu\gamma_0 - M\right]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\sigma}\psi)^2]$$

coupling constants

contact interactions

- Chiral symmetry realized and spontaneously broken as in QCD

- Chiral symmetry can be explicitly broken by the inclusion of small current quark masses.

Need to fix the coupling, the regularization scheme and the masses

No gauge dynamics (so no confinement)

No expansion parameter (hard to improve and keep control)

Chiral perturbation theory

A realisation of hadronic matter at low energy scales

$$p \ll \Lambda_\chi \sim 1\,\mathrm{GeV}$$

Qualitative recipe

Variationally derive the **nonperturbative vacuum** and **expand** around that vacuum for small momenta.

Since you are expanding, you have **control parameters**

We do not include baryons and vector mesons

$$|\mu_B| \lesssim 940 \text{ MeV} \qquad |\mu_I| \lesssim 770 \text{ MeV}$$

χPT@work

Fixing the low energy constants (LECs) in some way, we can make predictions

Example: pion scattering amplitude

$$T(\pi^+\pi^0 \to \pi^+\pi^0) = \frac{t - m_\pi^2}{f_\pi^2}$$

LECs

then the predict the amplitude for any other process like

$$\pi\pi \to 4\pi, 6\pi, 8\pi$$

NLO corrections can be computed in a systematic way

Powerful, but

Need to fix the LECs to be predictive

Need to know the degrees of freedom, symmetries + relevant energy scale

It lacks a microscopic description (at the quark level)

Leading order pion Lagrangian

The $\mathcal{O}(p^2)$ Lorentz invariant Lagrangian density for pseudoscalar mesons

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\nu \Sigma \partial^\nu \Sigma^\dagger) + \mathrm{Tr}(M\Sigma^\dagger + M^\dagger \Sigma)$$

low energy constants (LECs)

Meson field

SU(2)-flavor

$$\boxed{\Sigma = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \mathbf{1}_2 \cos\alpha \pm iN \sin\alpha}$$

$$N = \boldsymbol{n}\cdot\boldsymbol{\sigma}$$

$$n^1 = \sin\Theta\cos\Phi\ , \quad n^2 = \sin\Theta\sin\Phi\ , \quad n^3 = \cos\Theta\ ,$$

Pauli matrices

α, Θ, Φ three fields for three degrees of freedom

$\mathrm{SU}(2)$ **static and homogeneous vev**

$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$$

variational parameters

Static Lagrangian

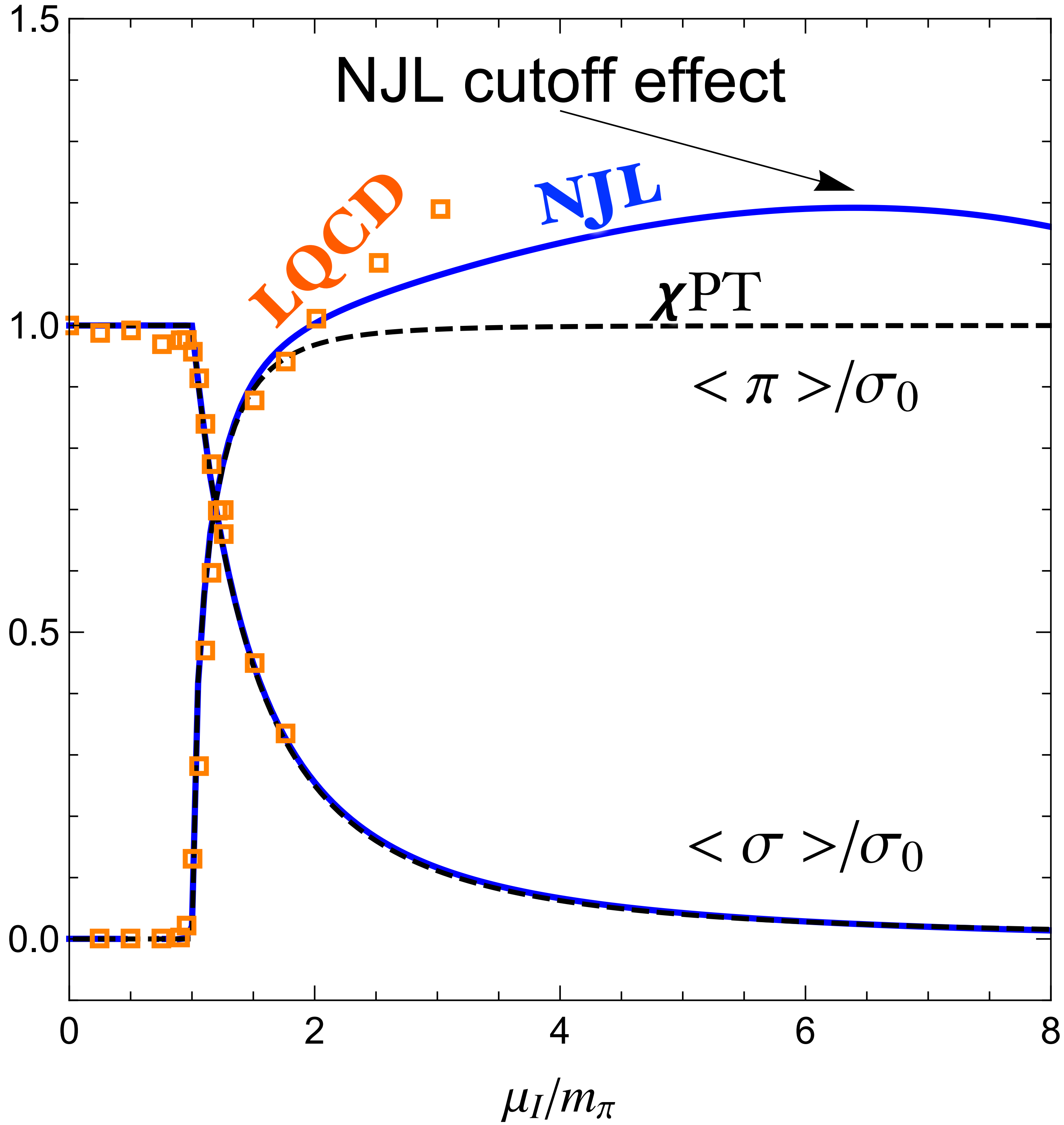
$$\mathcal{L}_0(\alpha, \mu_I, n_3) = F_0^2 m_\pi^2 \cos\alpha + \frac{F_0^2}{2}\mu_I^2 \sin^2\alpha(1 - n_3^2)$$

Maximising the Lagrangian

for $\mu_I < m_\pi$	$\cos\alpha = 1$	$\mathcal{L}_0$ independent of $\boldsymbol{n}$
for $\mu_I > m_\pi$	$\cos\alpha_\pi = m_\pi^2/\mu_I^2$	$n_3 = 0$ residual $O(2)$ symmetry

Comparing
LQCD, NJL and χPT

Pion and chiral condensates



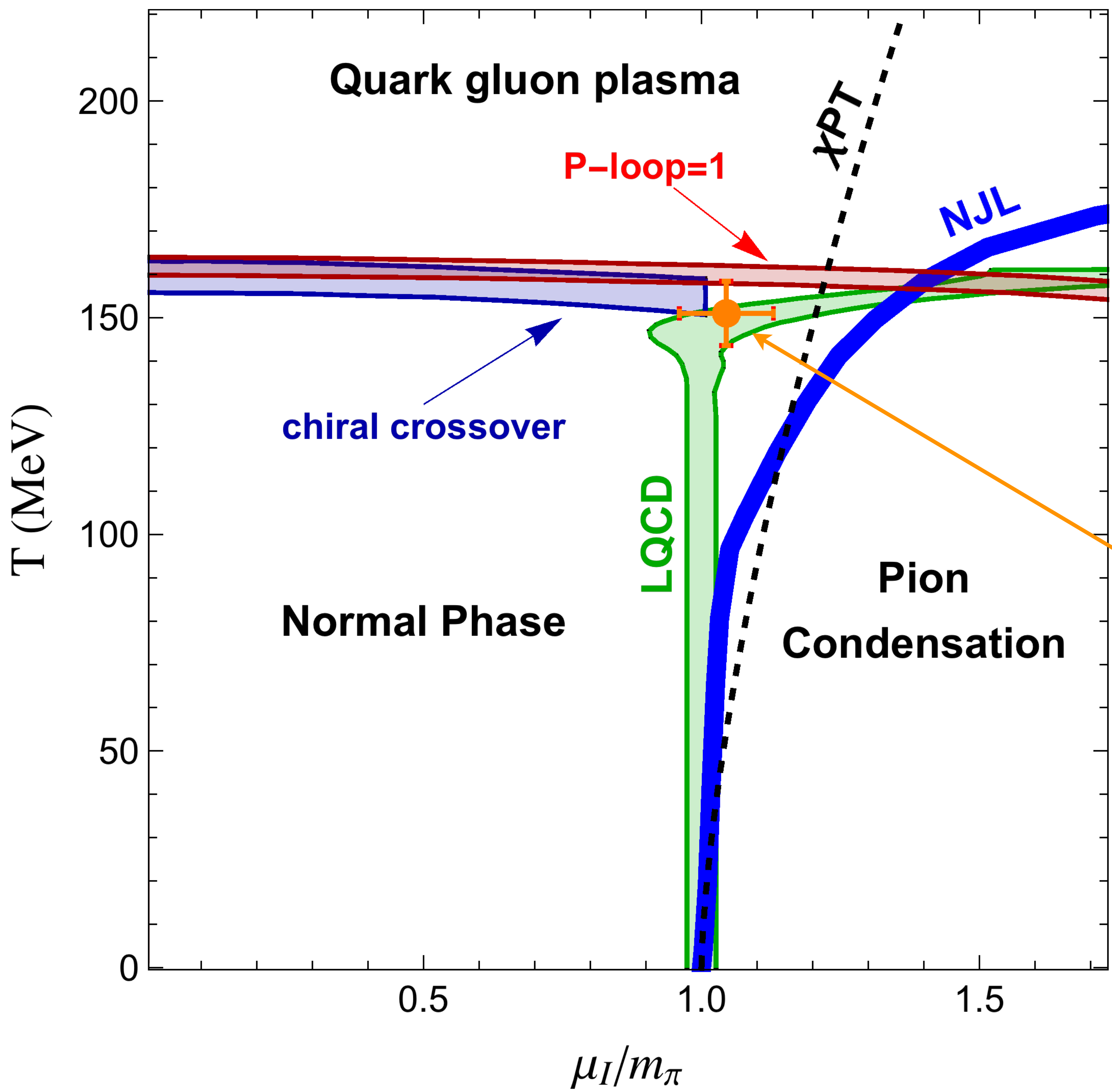
The three methods agree where they are supposed to work

More work to be done at large isospin

Brandt+ Phys. Rev. 2018, D 97, 054514

M.M. Particles 2 (2019) no.3, 411-443

Phase diagram



Qualitative similar behavior at low T

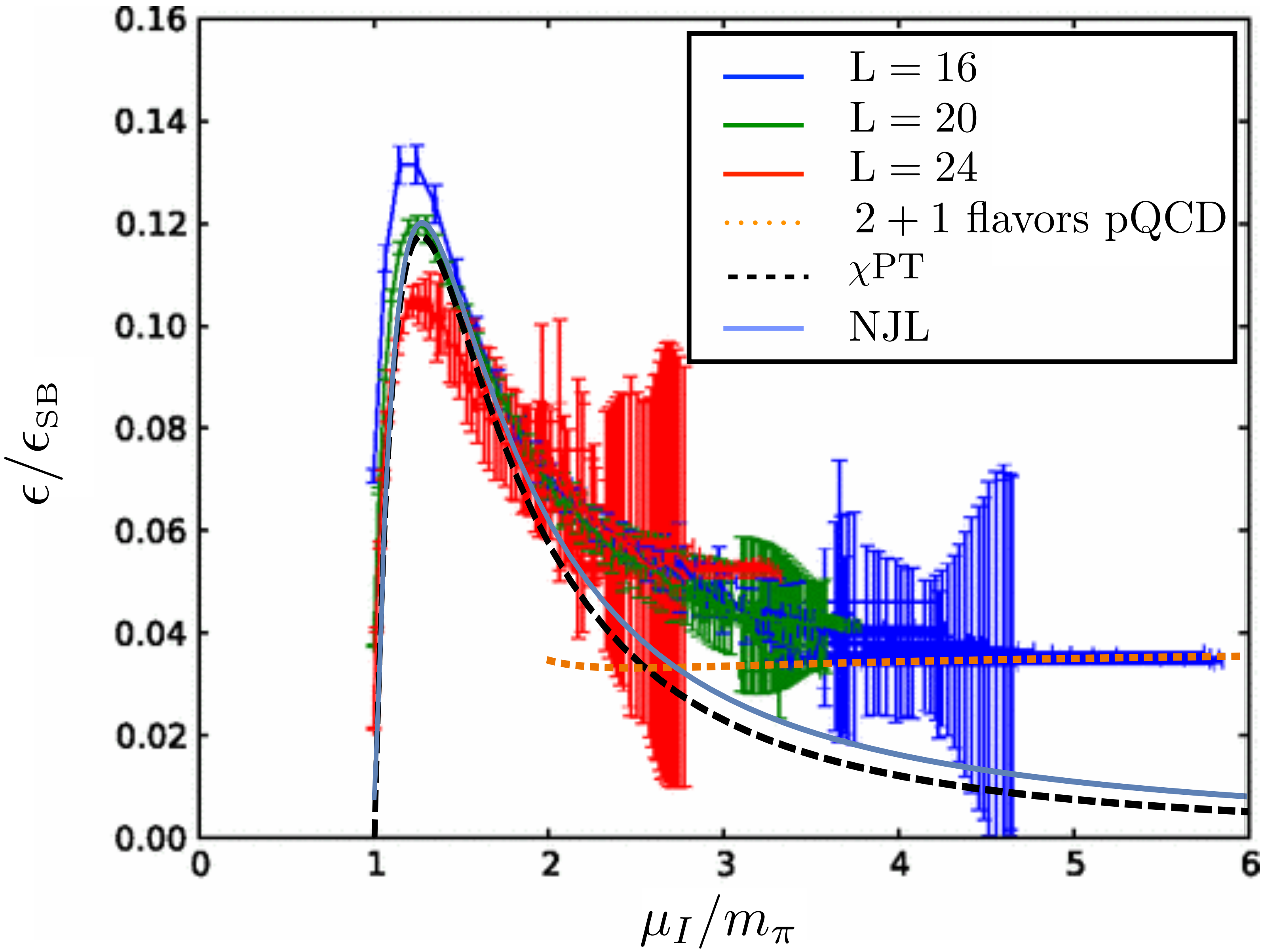
Not clear the origin of the discrepancies

(pseudo) tricritical point

Brandt+ Phys. Rev. 2018, D 97, 054514

M.M. Particles 2 (2019) no.3, 411-443

Results for the energy density



$$\epsilon_{SB} = \frac{N_c N_f}{4\pi^2}\mu_I^4$$

factor $\sim \frac{1}{16}$ missing

Canonical LQCD simulations

W. Detmold, K. Orginos, and Z. Shi, Phys. Rev. D86, 054507 (2012)

pQCD

T. Graf, et al. Phys. Rev. D 93, 085030 (2016)

χPT

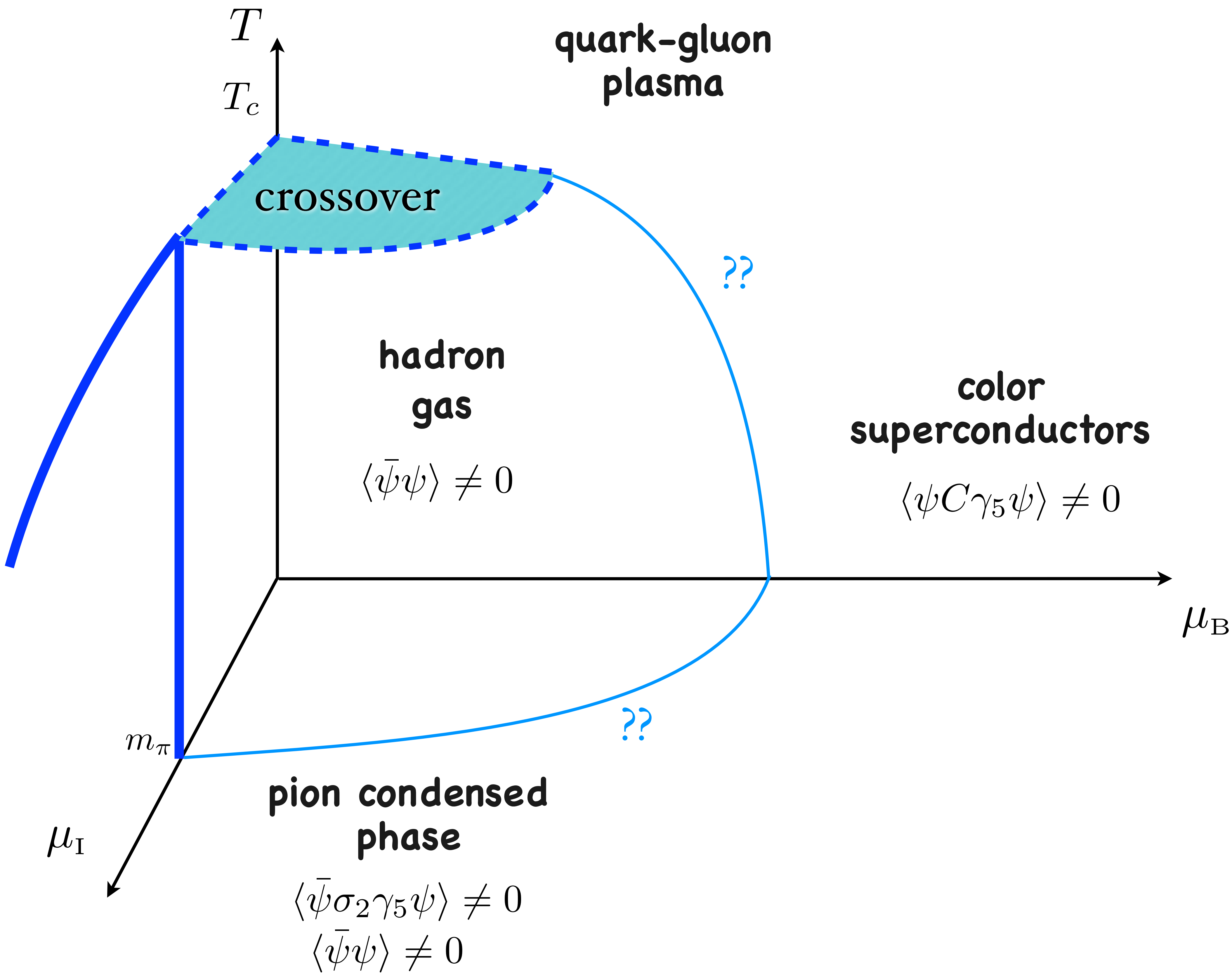
S. Carignano, A. Mammarella, MM Phys.Rev. D93 (2016) no.5, 051503

χPT gives an ANALYTIC expression for the peak

$\mu_{I,\rm LQCD}^{\rm peak} = \{1.20, 1.25, 1.275\} m_\pi$ $\quad\quad$ $\mu_{I,\chi\rm PT}^{\rm peak} = (\sqrt{13}-2)^{1/2} m_\pi \simeq 1.276\, m_\pi$

M.M. Particles 2 (2019) no.3, 411-443

Revisiting the QCD phase diagram



M.M. Particles 2 (2019) no.3, 411-443

Multimeson condensation

L. Lepori and M.M. Phys.Rev. D99 (2019) no.9, 096011

Multicomponent superfluids

Systems with **simultaneous** condensation of two species

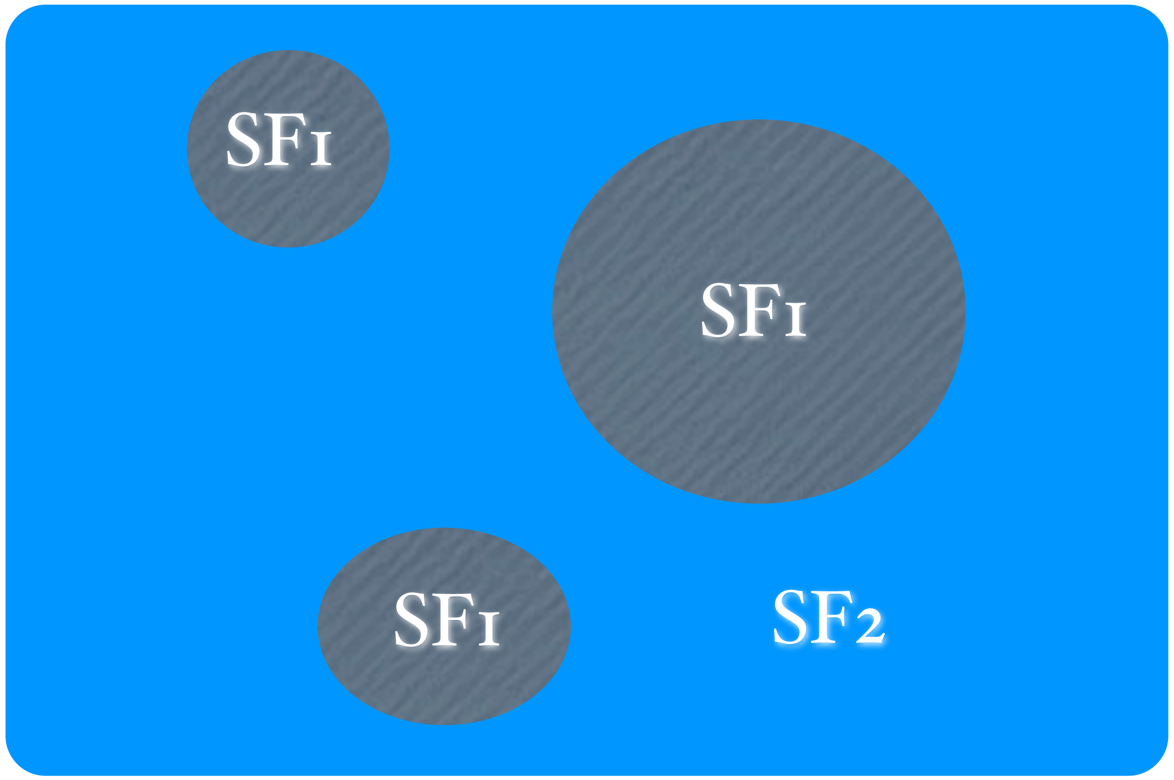
- Mixtures as $^{4}\mathrm{He}$ - $^{3}\mathrm{He}$ can coexist as superfluids
- In compact stars neutrons and protons can be simultaneously superfluid

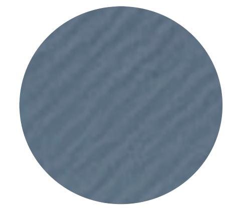
SF: single superfluid

SCO: Simultaneous condensation

We will explore whether something similar can happen with mesons

Multimeson condensation





Pion superfluid bubbles

Kaon superfluid see

SF: single superfluid

Or they may coexist

SCO: Simultaneous condensation

Mutimeson systems

Suppose that we have two noninteracting meson gases.

Symmetry group $\quad G = G_1 \times G_2 \quad$ with $\quad G_a = \{SU(N_f)_L \times SU(N_f)_R\}_a$

Indeed, I can arbitrarily rotate the two meson fields

$$\Sigma_1 \to L_1 \Sigma_1 R_1^\dagger \qquad \text{and} \qquad \Sigma_2 \to L_2 \Sigma_2 R_2^\dagger$$

LO chiral Lagrangian with no intraspecies interactions

$$\begin{aligned}\mathcal{L} =& \frac{f_{1\pi}^2}{4} \mathrm{Tr}(D_\nu^1 \Sigma_1 D^{1\nu} \Sigma_1^\dagger) + \mathrm{Tr}(\Sigma_1 M_1^\dagger + M_1 \Sigma_1^\dagger) \\ &+ \frac{f_{2\pi}^2}{4} \mathrm{Tr}(D_\nu^2 \Sigma_2 D^{2\nu} \Sigma_2^\dagger) + \mathrm{Tr}(\Sigma_2 M_2^\dagger + M_2 \Sigma_2^\dagger)\end{aligned}$$

Interaction terms

Which are the possible interaction terms?

Two possibilities:

"Unlock" interaction

$$\mathcal{L}_{\text{int,unlock}} = \tilde{L}_1 \mathrm{Tr}(D^1_\mu \Sigma_1 D^{1\mu} \Sigma_1^\dagger) \mathrm{Tr}(D^2_\nu \Sigma_2 D^{2\nu} \Sigma_2^\dagger) + \tilde{L}_2 \mathrm{Tr}(D^1_\mu \Sigma_1 D^{1\nu} \Sigma_1^\dagger) \mathrm{Tr}(D^2_\mu \Sigma_2 D^{2\nu} \Sigma_2^\dagger)$$

$G = G_1 \times G_2$

unbroken

$\mathcal{O}(p^4)$ in χPT: subleading

"Lock" interaction

$$\mathcal{L}_{\text{int,lock}} = k \frac{f_{1\pi} f_{2\pi}}{2} \mathrm{Tr}(D^1_\nu \Sigma_1 D^{2\nu} \Sigma_2^\dagger)$$

$G = G_1 \times G_2$

broken to

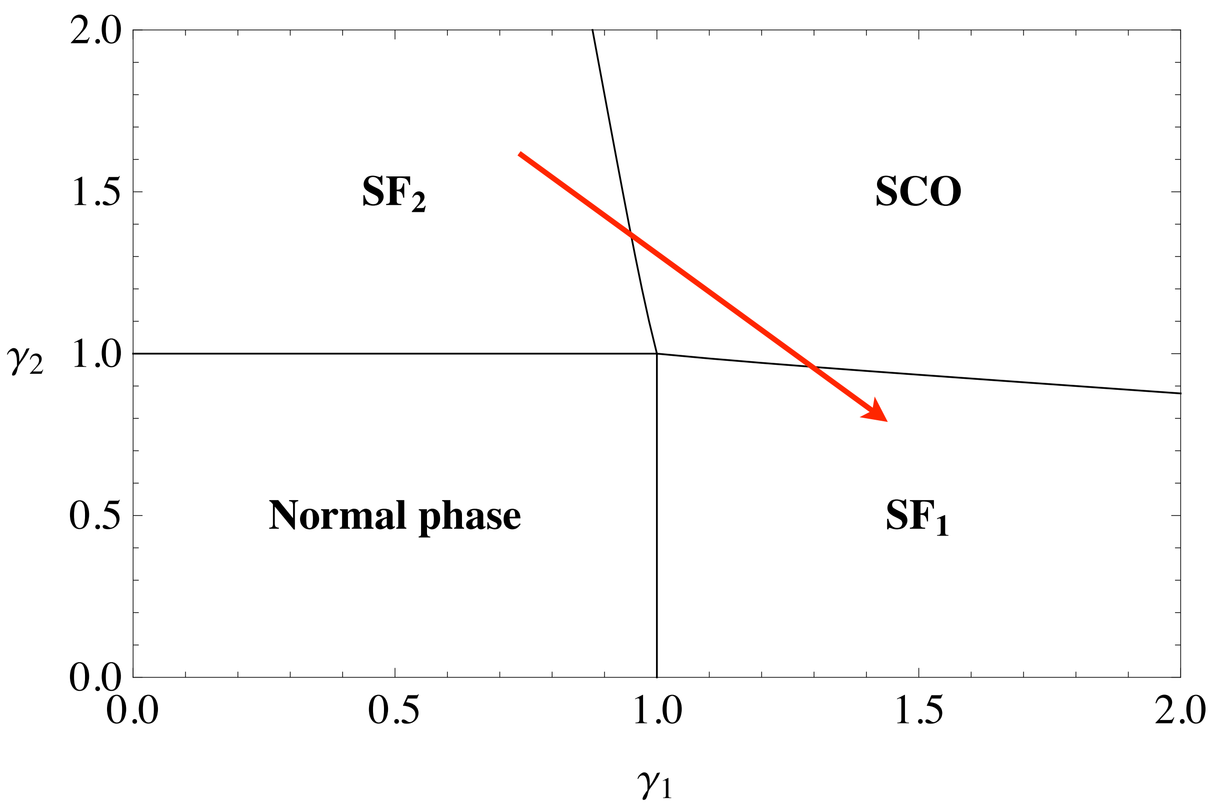
$G_D = SU(N_f)_L \times SU(N_f)_R$

$\mathcal{O}(p^2)$ in χPT: Leading order!

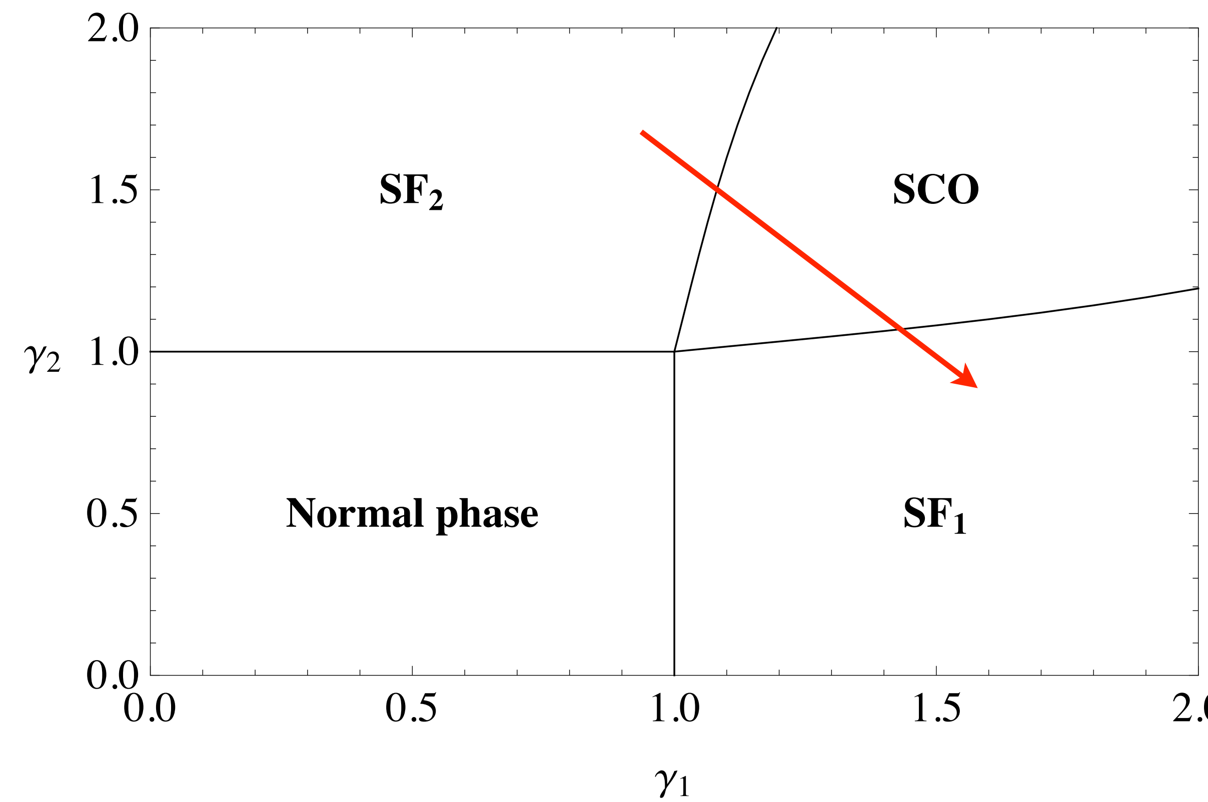
Two pion gases: "Unlock" interaction

Control parameters $\gamma_i = \frac{\mu_i}{m_\pi}$

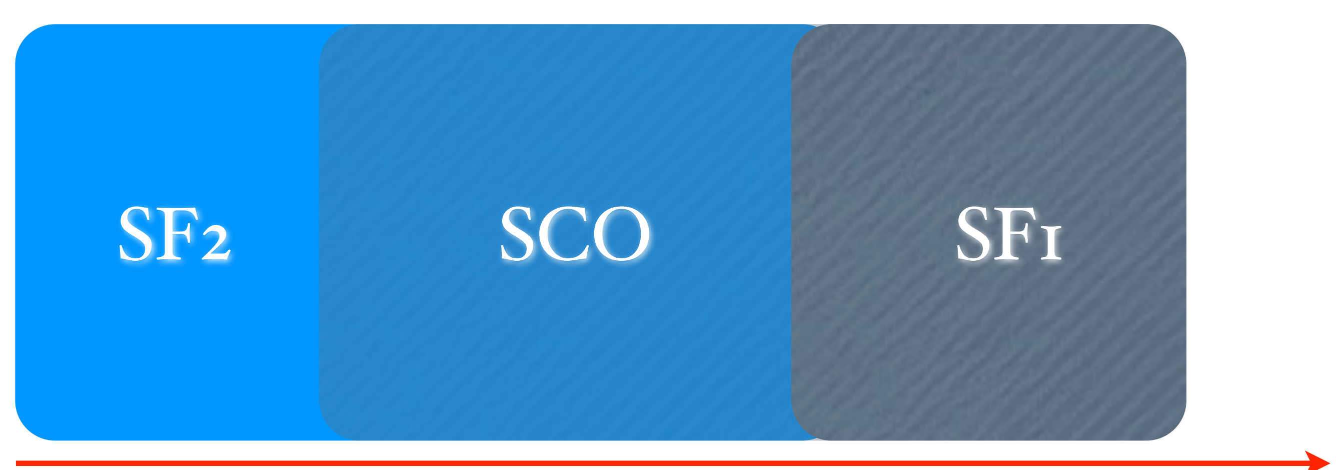
$\tilde{L}_1 + \tilde{L}_2 = +10^{-2}$



$\tilde{L}_1 + \tilde{L}_2 = -10^{-2}$

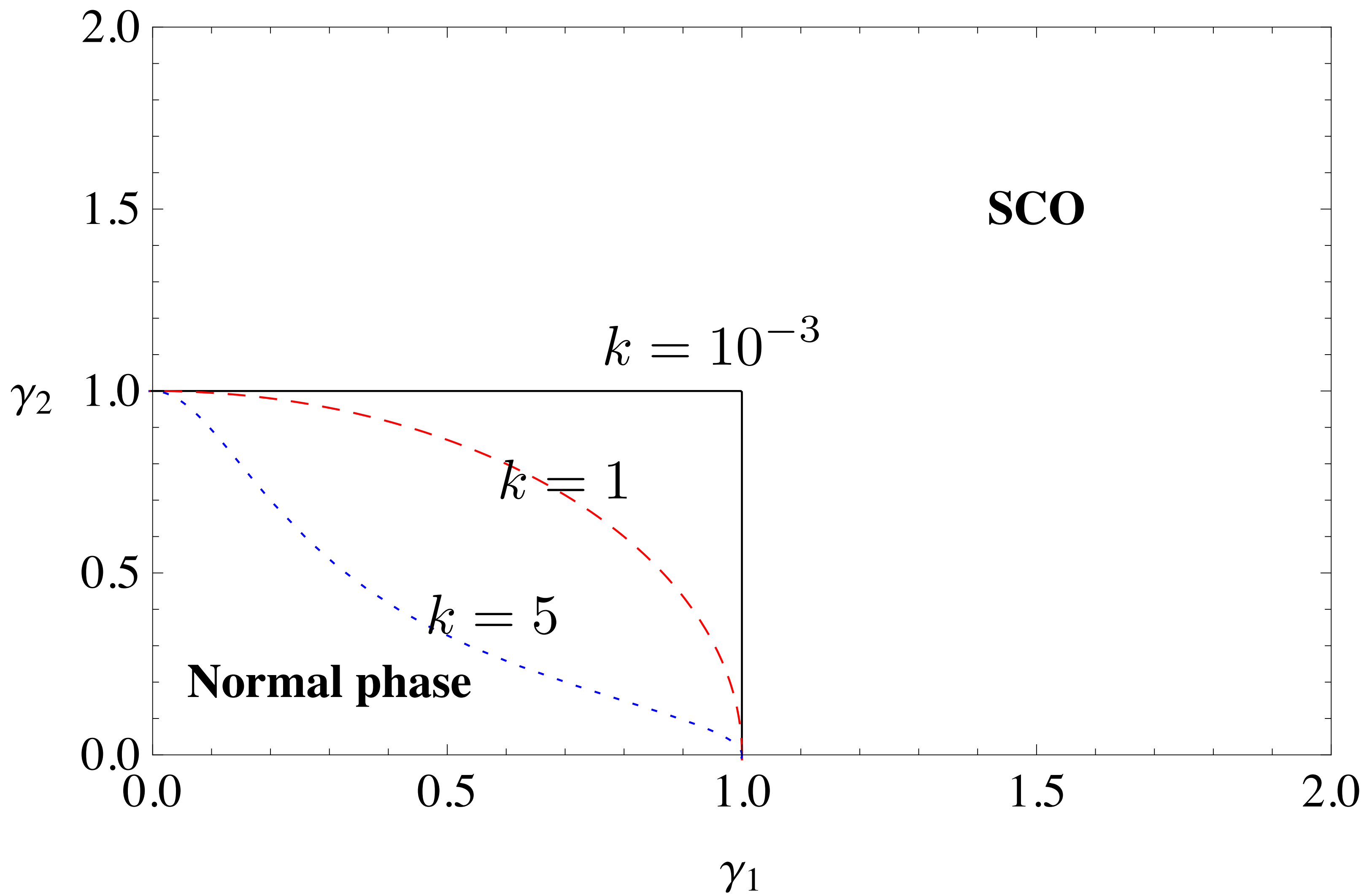


SCO: Simultaneous condensation
SF: Single superfluid



Two pion gases: "Lock" interaction

Control parameters $\gamma_i = \dfrac{\mu_i}{m_\pi}$



With increasing interaction strength the normal phase shrinks...
eventually to zero?

Scrutinizing the locking interaction

Normal phase

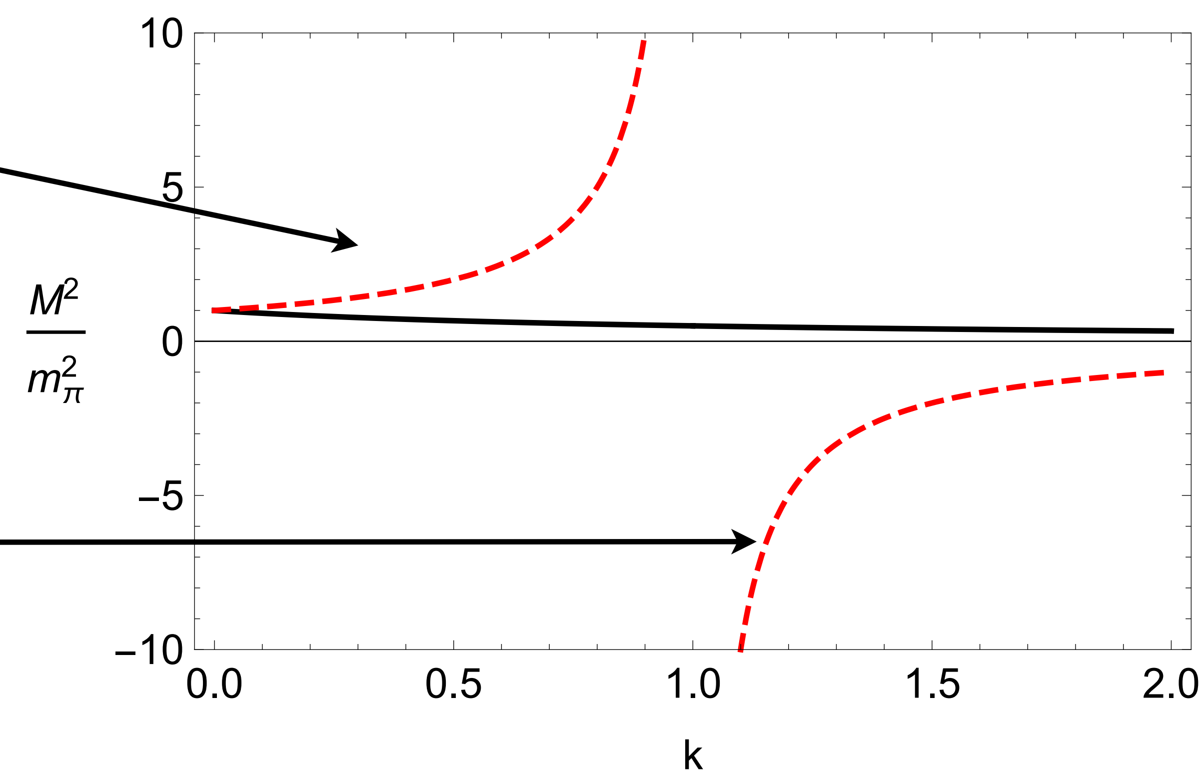
$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\nu \Sigma_1 \partial^\nu \Sigma_1^\dagger) + \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\nu \Sigma_2 \partial^\nu \Sigma_2^\dagger) + \frac{f_\pi^2 m_{1\pi}^2}{2}\mathrm{Tr}(\Sigma_1) + \frac{f_\pi^2 m_{2\pi}^2}{2}\mathrm{Tr}(\Sigma_2) + k\frac{f_\pi^2}{2}\mathrm{Tr}(\partial_\nu \Sigma_1 \partial^\nu \Sigma_2^\dagger)$$

two parameters: not the masses!

Masses $$M_\pm^2 = \frac{m_{1\pi}^2 + m_{2\pi}^2 \pm \sqrt{(m_{1\pi}^2 - m_{2\pi}^2)^2 + 4k^2 m_{1\pi}^2 m_{2\pi}^2}}{2(1-k^2)}$$

Mass splitting

Instability!



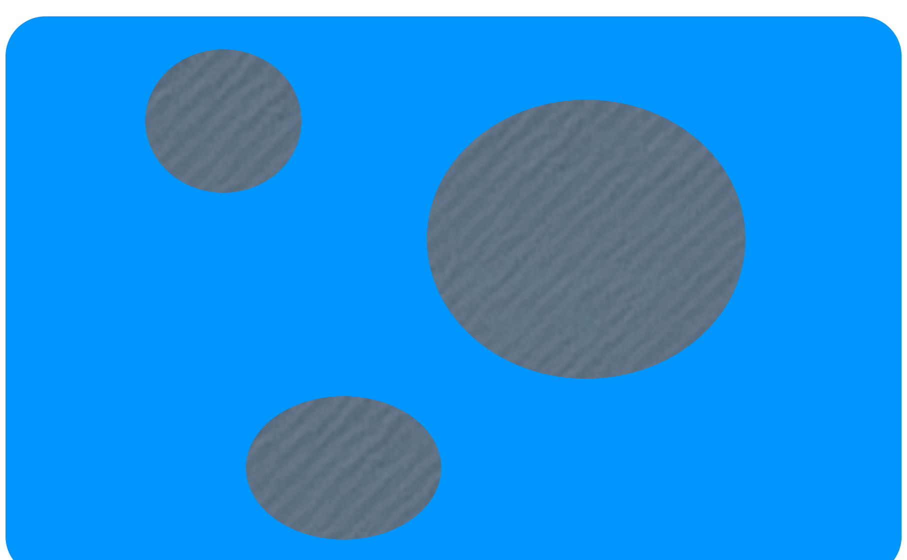
Toy model

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}m_1^2\phi_1^2}_{\text{free field 1}} + \underbrace{\frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}m_2^2\phi_2^2}_{\text{free field 2}} + \underbrace{k\,\partial_\mu\phi_1\partial^\mu\phi_2}_{\text{interaction}}$$

Symmetry breaking $\quad Z_2 \times Z_2 \to Z_2$

Unstable for |k|>1 because one of the two eigenmodes has an imaginary mass

Maybe solution of the instability by the realization of an inhomogeneous phase



$$\mathcal{S} = \int d^4x\,\mathcal{L} \approx \int_{V_1} d^4x\,\mathcal{L}_1 + \int_{V_2} d^4x\,\mathcal{L}_2 + \int_{S_{12}} d^4x\,\mathcal{L}$$
$$= \mathcal{S}_1 + \mathcal{S}_2 + S_{\text{interface}}$$

Solitonic phase

F. Canfora, S. Carignano, M. Lagos, MM, A. Vera work in progress

Classical field solution

Solitons correspond to time-independent nontrivial classical field solution

Restart from the unimodular field

$$\Sigma = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \mathbf{1}_2 \cos\alpha \pm iN \sin\alpha$$

where $N = \boldsymbol{\sigma}\cdot\boldsymbol{n}$ and $n^1 = \sin\Theta\cos\Phi\ ,\quad n^2 = \sin\Theta\sin\Phi\ ,\quad n^3 = \cos\Theta\ ,$

α, Θ, Φ promoted to classical fields

Classical equations of motion

$$\partial_\mu \partial^\mu \Phi = -(\partial_\mu \Phi - \mu_I \delta_{\mu_0})\partial^\mu(\sin^2\alpha \sin^2\theta)$$

$$\partial_\mu \partial^\mu \Theta = \frac{\sin 2\Theta}{2}(\partial_\mu \Phi - \mu_I \delta_{\mu_0})(\partial^\mu \Phi - \mu_I \delta^{\mu_0})$$

$$\partial_\mu \partial^\mu \alpha = -m_\pi^2 \sin\alpha + \frac{\sin(2\alpha)}{2}(\partial_\mu \Theta \partial^\mu \Theta + \sin^2(\Theta)(\partial_\mu \Phi - \mu_I \delta_{\mu 0})(\partial^\mu \Phi - \mu_I \delta^{\mu 0}))$$

Only derivatives of Φ appear: it is the NGB

In inhomogeneous solutions we expect

$\alpha = 0$ Normal phase

$\alpha = \bar{\alpha} \neq 0$ Broken phase

To have a solitonic stable solution we also demand that

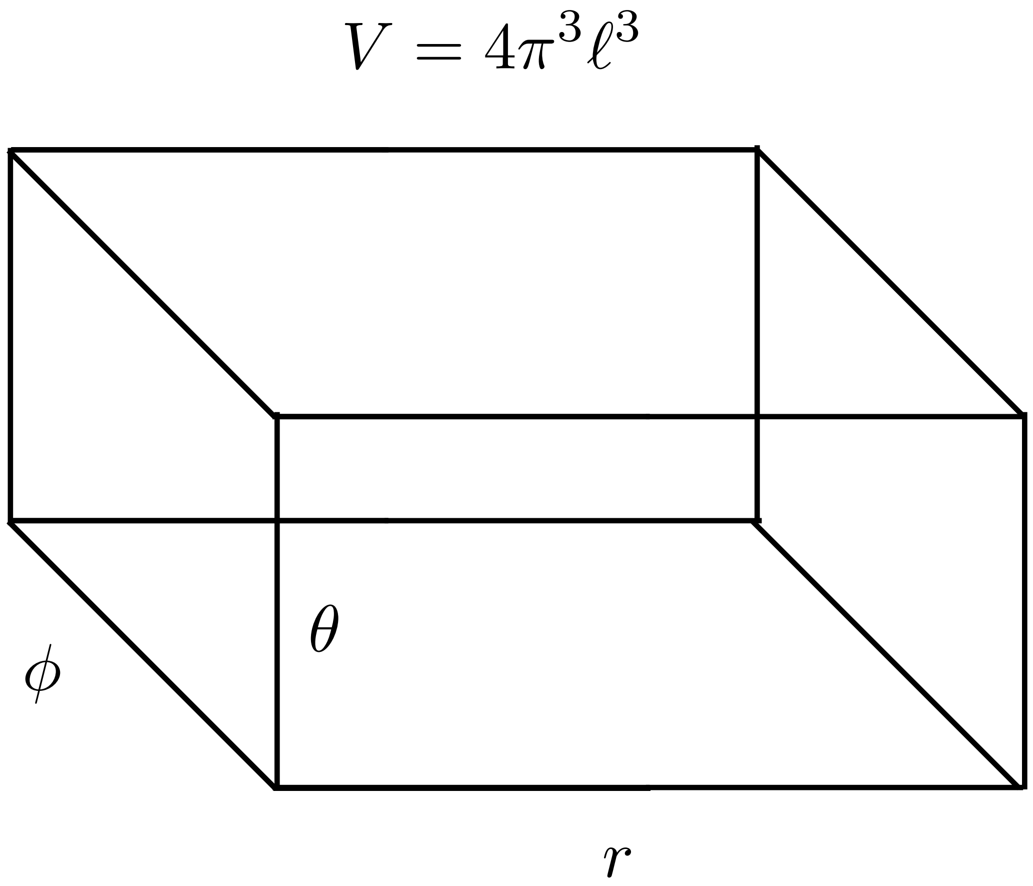
α, Θ, Φ Space dependent

Boundary conditions

Finite size system

$$ds^2 = dt^2 - \ell^2 \left(dr^2 + d\theta^2 + d\phi^2\right)$$

$$0 \leq r \leq 2\pi \ , \quad 0 \leq \theta \leq \pi \ , \quad 0 \leq \phi \leq 2\pi \ ,$$



Dirichlet boundary conditions

$$\Sigma(0,\theta,\phi) = \Sigma(2\pi,\theta,\phi)$$
$$n(r,0,\phi) = -n(r,\pi,\phi)$$
$$n(r,\theta,0) = n(r,\theta,2\pi)$$

different BCs can be easily implemented

Decoupling

$$\partial_\mu \partial^\mu \Phi = -\,(\partial_\mu \Phi - \mu_I \delta_{\mu_0})\partial^\mu (\sin^2 \alpha \sin^2 \theta)$$

vanishes

$$\Phi \equiv \Phi(t, \phi)$$

$$\alpha \equiv \alpha(r, \theta) \quad \Theta \equiv \Theta(r, \theta)$$

$$\partial_\mu \partial^\mu \Phi = \left(\frac{\partial^2}{\partial t^2} - \frac{1}{\ell^2} \frac{\partial^2}{\partial \phi^2} \right) \Phi = 0$$ free field equation

Two possible solutions

$$\Phi \propto e^{i\omega t - k\ell\phi}$$ dynamical field

$$\Phi = a\frac{t}{\ell} - p\,\phi + \Phi_0$$ solitonic-like solution

$p \in \mathbb{Z}$ to match boundary conditions

Looking for solitons

$$\Phi \equiv \Phi(t,\phi)$$

$$\alpha \equiv \alpha(r,\theta) \qquad \Theta \equiv \Theta(r,\theta)$$

$$\nabla^2\Theta = -K\frac{\sin 2\Theta}{2}$$

where $\quad K = (\partial_\mu\Phi - \mu_I\delta_{\mu_0})(\partial^\mu\Phi - \mu_I\delta^{\mu_0}) = (a/\ell - \mu_I)^2 - p^2 \quad$ constant

sine-Gordon-like equation with $\quad \nabla^2 = \frac{1}{\ell^2}\left(\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial r^2}\right)$

$$\Theta(\bar\theta,\bar r) = 2\arctan\left[\frac{\sinh(u\bar\theta/\sqrt{u^2-1})}{u\cos(\bar r/\sqrt{u^2-1})} + \delta\right]$$

$$\bar r = \ell r K/2 \qquad \bar\theta = \ell\theta K/2$$

rescaling

constants depending on BCs

$u \qquad \delta$

$$\Theta(0,r) = 0 \longrightarrow \delta = 0$$

$$\Theta(\pi,r) = (2k+1)\pi \longrightarrow K = 0$$

$\Theta = q\theta + \Theta_0 \qquad q \quad$ odd

Modulation

$$\Phi = a\frac{t}{\ell} - p\,\phi + \Phi_0$$

$$\Theta = q\theta + \Theta_0$$

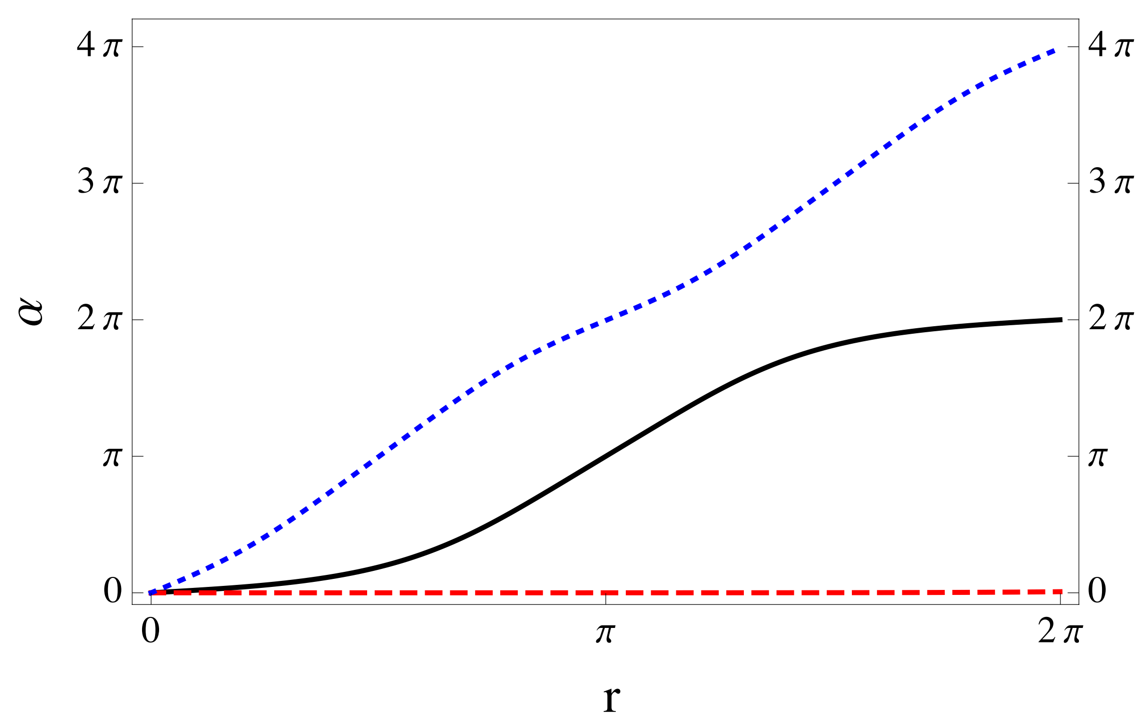
$$\boxed{\frac{\partial^2\alpha}{\partial r^2} = m_\pi^2\ell^2 \sin\alpha + \frac{q^2}{2}\sin(2\alpha)}$$

Integrated using $\frac{\partial\alpha}{\partial r} = \eta(\alpha)$ $\quad \eta(\alpha) = \pm\sqrt{\eta_0^2 + 2m_\pi^2\ell^2(1-\cos\alpha) + q^2\sin^2(\alpha)}$

Integrating $\int_0^{2n\pi} \frac{d\alpha}{\eta(\alpha)} = 2\pi$

Fix η_0 such that

$\alpha(2\pi) = 2n\pi$



Topological charge

$$B = \frac{\ell^3}{24\pi^2} \int_S dr d\theta d\phi \ \rho_{\mathrm{m}}$$

where $\quad \rho_{\mathrm{m}} = \epsilon^{ijk} \mathrm{Tr} \left\{ \left(\Sigma^{-1} \partial_i \Sigma \right) \left(\Sigma^{-1} \partial_j \Sigma \right) \left(\Sigma^{-1} \partial_k \Sigma \right) \right\}$

using the previous results $\quad \rho_{\mathrm{m}} = \frac{3pq}{\ell^3} \sin(q\theta) \frac{\partial}{\partial r} \left(\sin(2\alpha) - 2\alpha \right) ,$

$$\boxed{B = \begin{cases} -2p & \text{if} \quad q \text{ odd} \\ 0 & \text{if} \quad q \text{ even} \end{cases}}$$

The topological charge protects the soliton from decay

Conclusions

- We can attack the QCD phase diagram from various different sides

- The isospin chemical potential side gives less resistance to the theoretical/numerical attack.

- Pion condensation is an important path **for understanding some aspects of QCD**

- The comparison between LQCD simulations, NJL modeling and Chiral perturbation theory corroborates the obtained results and shows the limitations

- Is it possible to realize inhomogeneous phases in LQCD?

back up

Leading order results at T=0

From the static Lagrangian it is possible to determine all static properties

Pressure $$p = \frac{f_\pi^2 \mu_I^2}{2}\left(1 - \frac{m_\pi^2}{\mu_I^2}\right)^2$$

Number density $$n_I = \frac{\partial p}{\partial \mu_I} = f_\pi^2 \mu_I \left(1 - \frac{m_\pi^4}{\mu_I^4}\right)$$

Equation of State

$$\epsilon(p) = 2\sqrt{p(2f_\pi^2 m_\pi^2 + p)} - p$$

S. Carignano, A. Mammarella and M.M.
Phys.Rev. D93 (2016) no.5, 051503

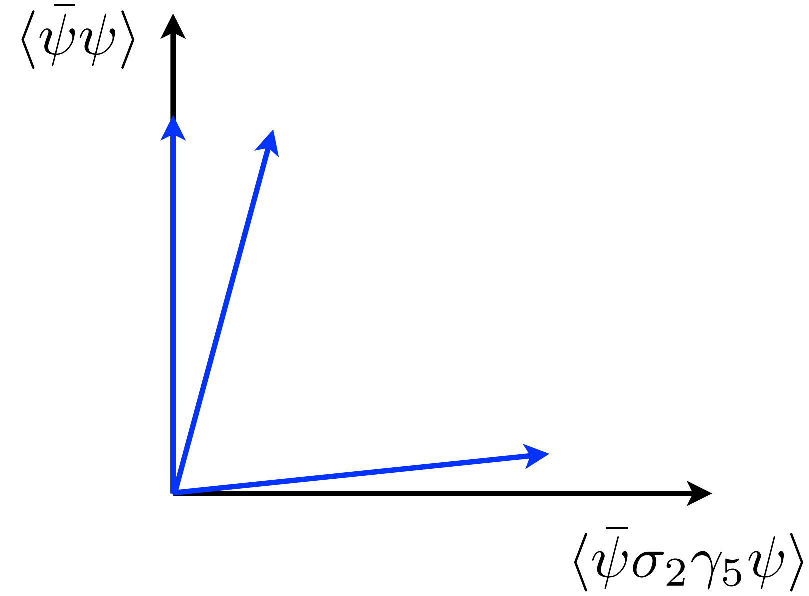
Depicting the pion condensation

$\mu_I = 0$

$\mu_I < m_\pi$

$\mu_I > m_\pi$

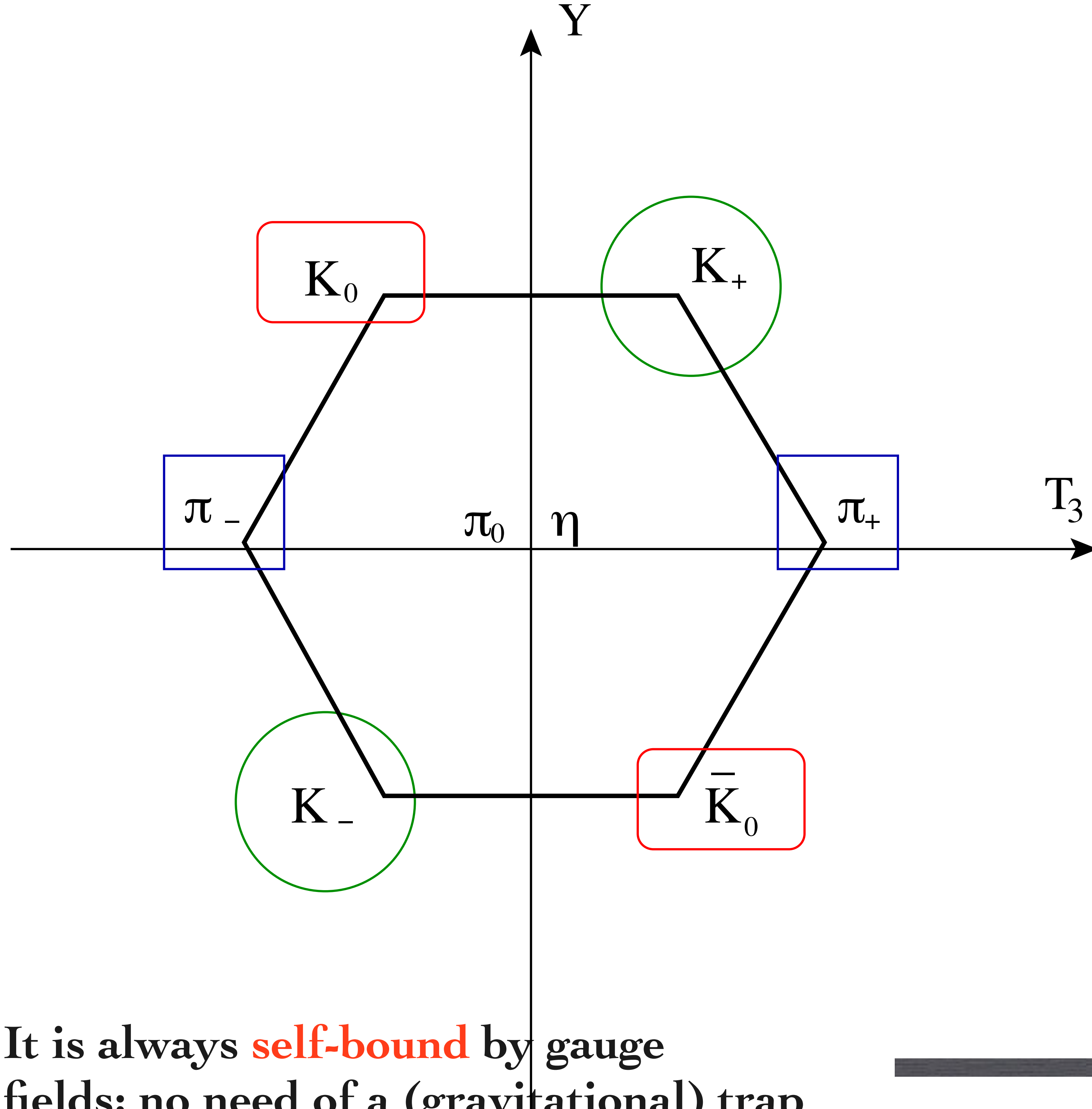
$\mu_I \gg m_\pi$



The condensate is “rotated”

Scalar condensate $\langle\bar u u\rangle = \langle\bar d d\rangle \propto \cos\alpha$

Pseudo scalar condensate $\langle\bar d\gamma_5 u + \text{h.c.}\rangle \propto \sin\alpha$



1. It is always self-bound by gauge fields: no need of a (gravitational) trap → "Bag" constant

2. The interaction between quarks is nonperturbative → Need some modeling

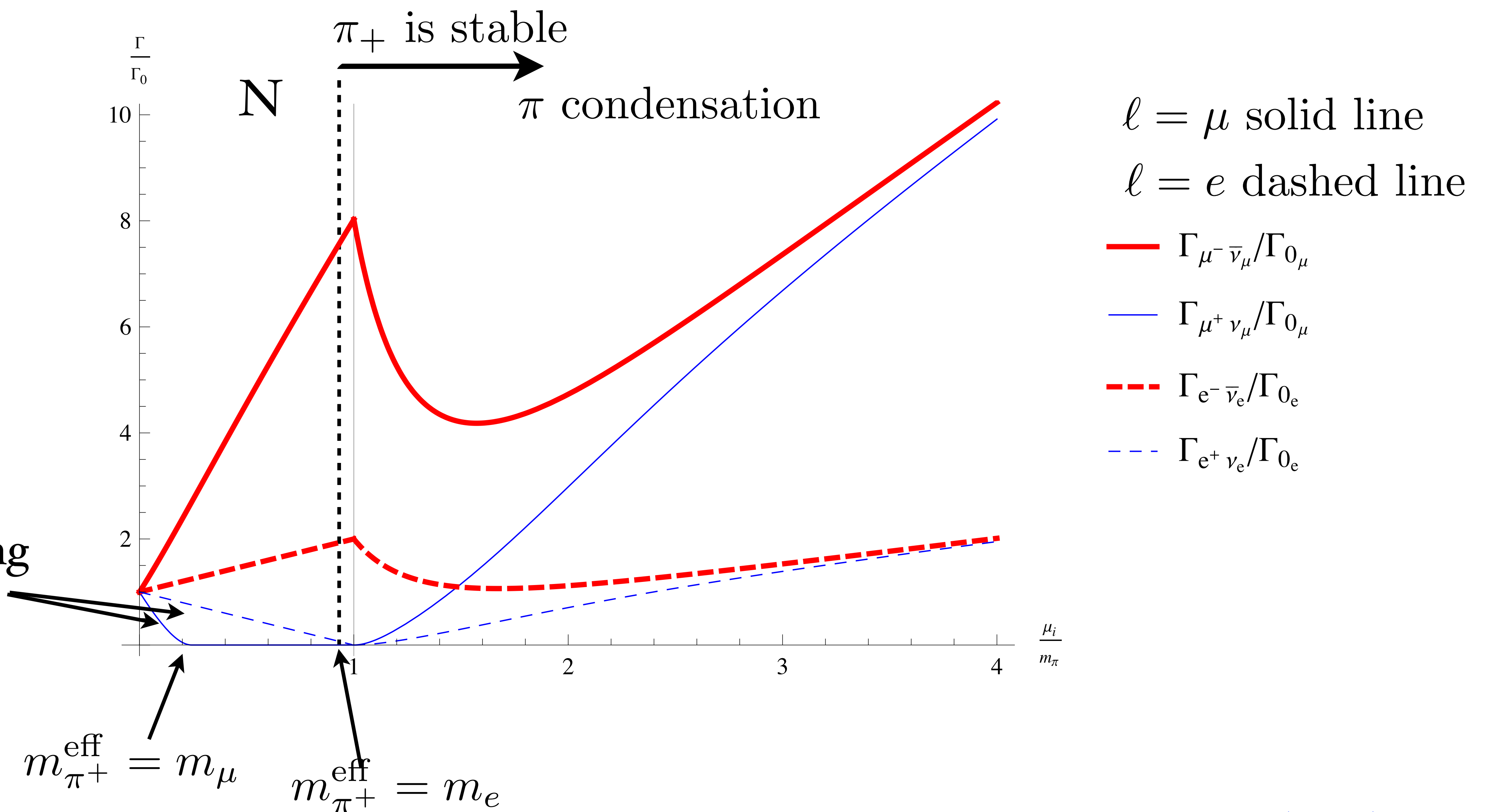
Leptonic decays

It is instructive to think of π_1 and π_2 as fundamental fields.

$$\begin{array}{ll} \mu_I < m_\pi & \pi_1 = \pi^+ \\ \mu_I > m_\pi & \pi_1 = \cos\theta\,\pi^+ + \sin\theta\,\pi^- \end{array}$$

$$\begin{array}{ll} \mu_I < m_\pi & \pi_2 = \pi^- \\ \mu_I > m_\pi & \pi_2 = \cos\theta\,\pi^- - \sin\theta\,\pi^+ \end{array}$$

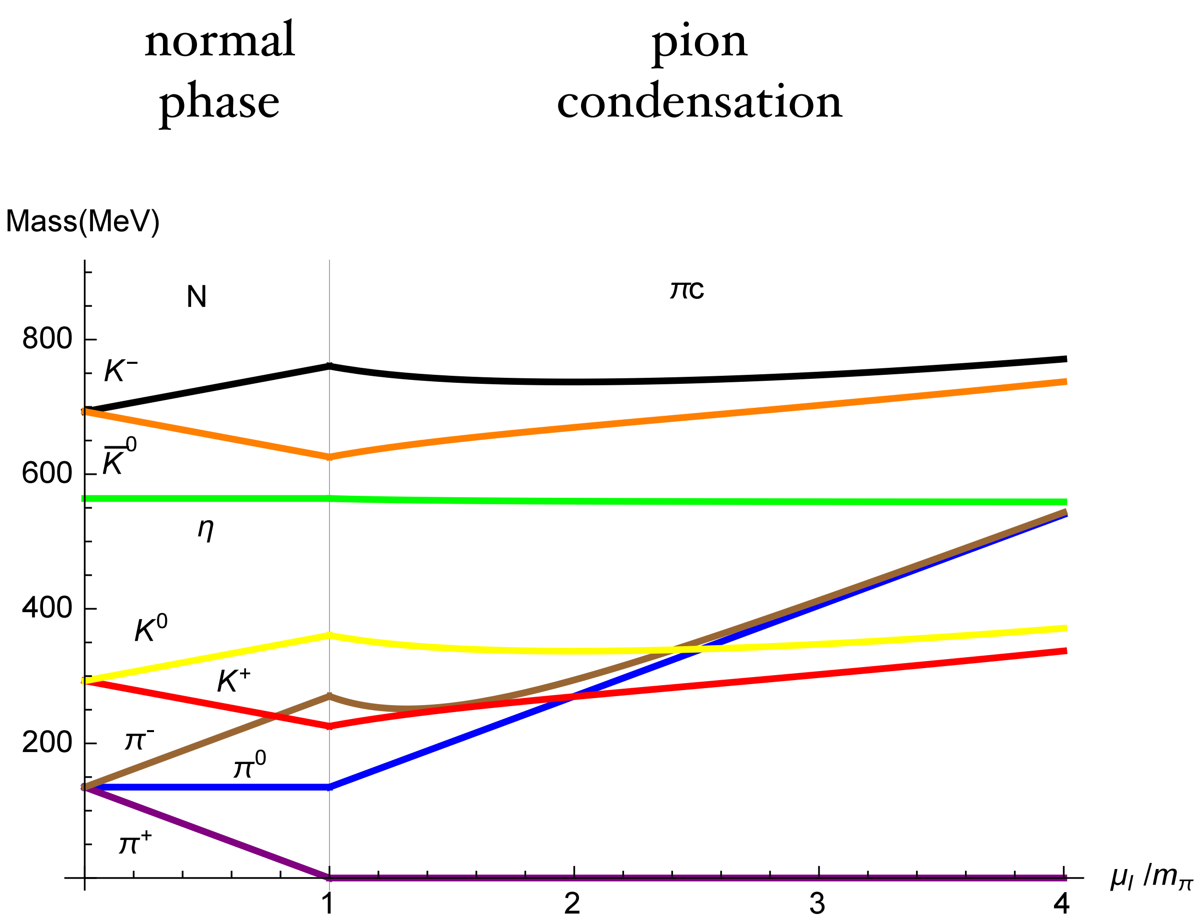
Processes $\pi_1 \to \ell^\pm \nu_\ell$ and $\pi_2 \to \ell^\pm \nu_\ell$

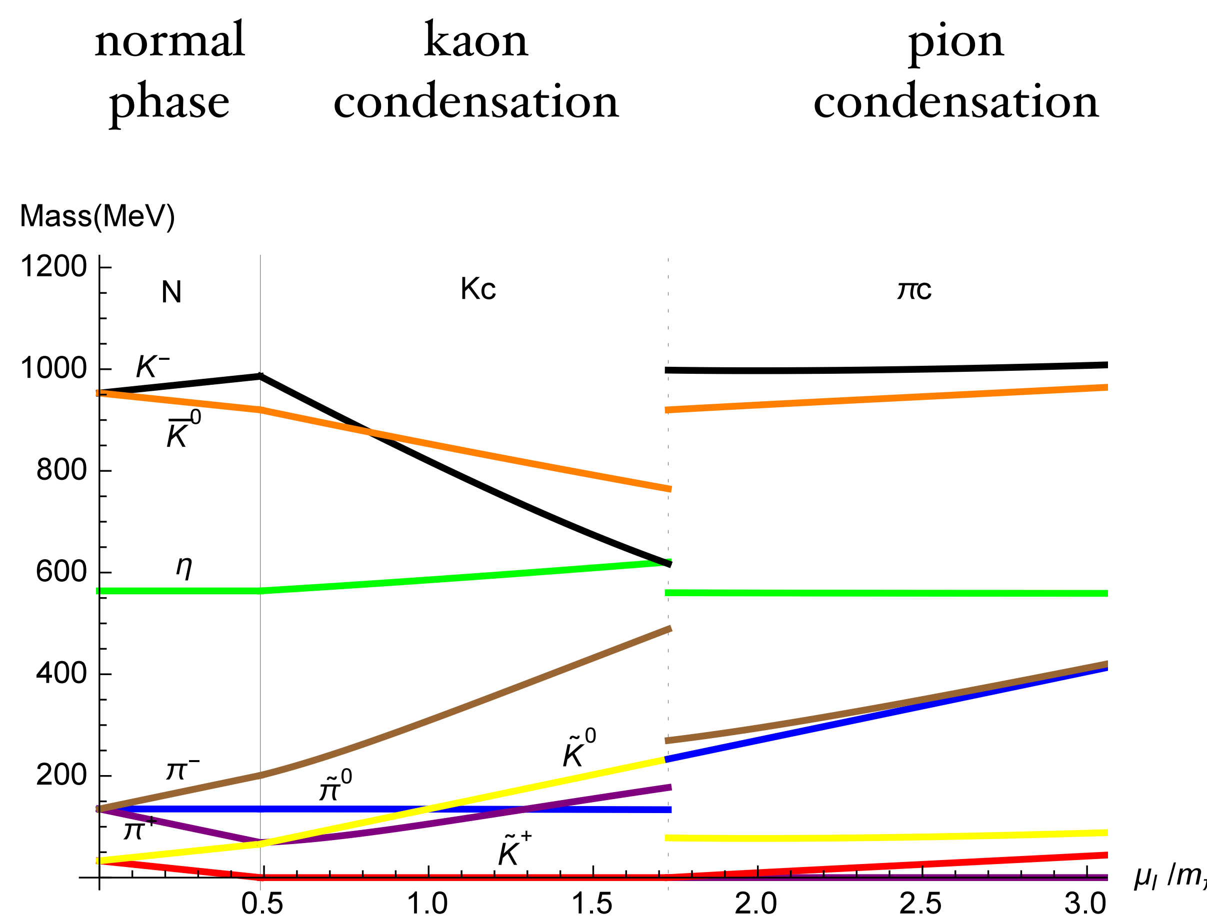


A. Mammarella and M.M. Phys.Rev. D92 (2015) 8, 085025

Mixing and mass splitting

In the condensed phases mesons mix and nontrivial mass splitting





$$\begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \pi_+ \\ \pi_- \end{pmatrix}$$

mass
eigenstates

charge
eigenstates

A. Mammarella and M.M. Phys.Rev. D92 (2015) 8, 085025

NLO corrections and LECS

It seems that we have the opportunity to infer the NLO chiral LECS

$$m_{\pi,4}^2 = m_\pi^2(1 + 16c - 8b)$$
$$f_{\pi,4}^2 = f_\pi^2(1 + 8b)$$

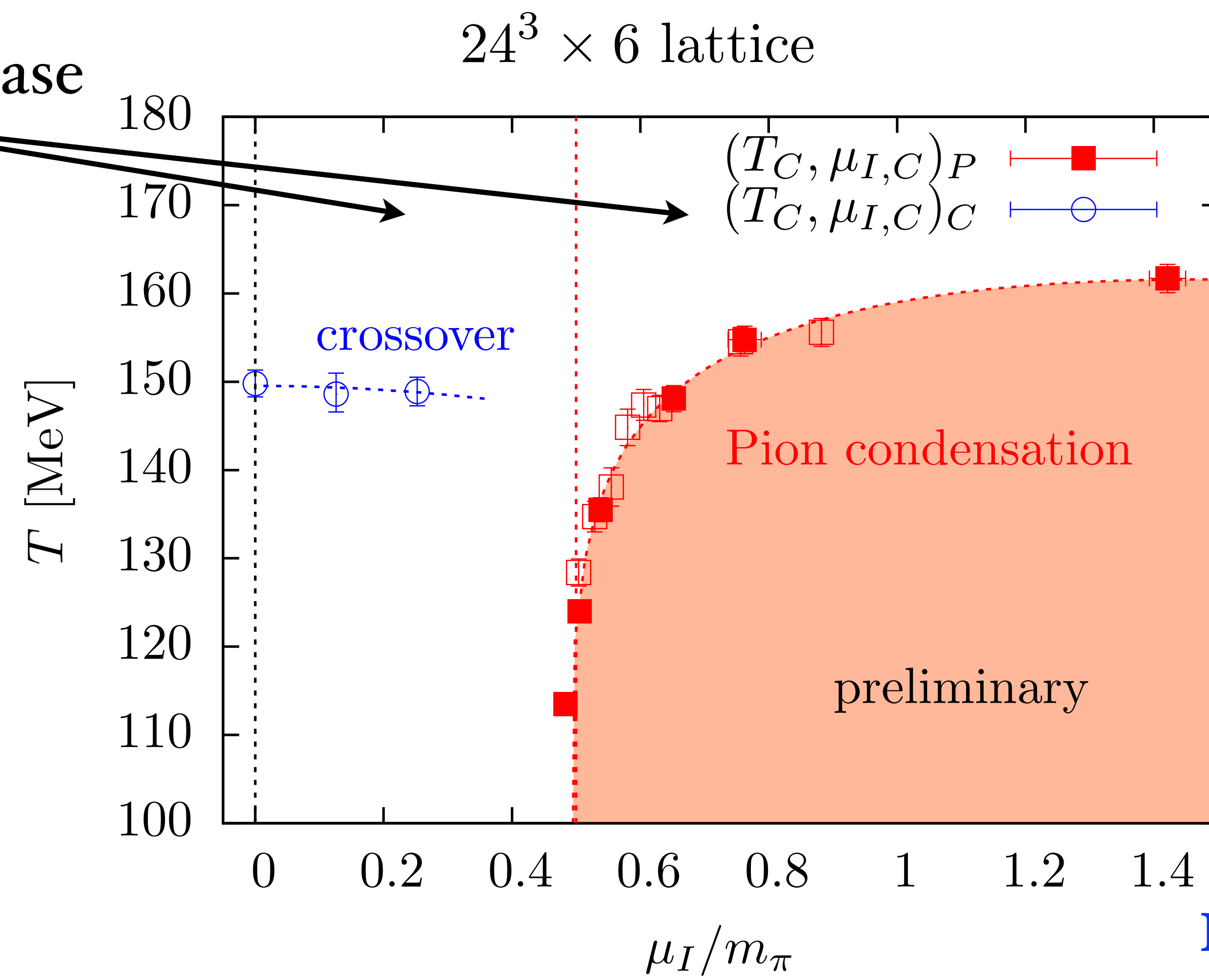
where a, b and c are of order 10^{-3} and are combinations of the SU(2) LECS

More refined lattice data are needed.

Extension of the present results to finite temperature

Finite temperature

chirally restored phase



$$\mu_I \to \mu_I/2$$

Brandt and Endrodi 1611.06758

The transition to **the pion condensed phase is a phase transition**

Therefore it can be quantitatively better studied than **the chiral restoration crossover**

Alternative descriptions

Why is the theory so complicated?
Pions are no more charge conjugate fields, they mix etc..

At the lowest order in derivatives and close to the phase transition mapping to a Gross-Pitaevskii Lagrangian

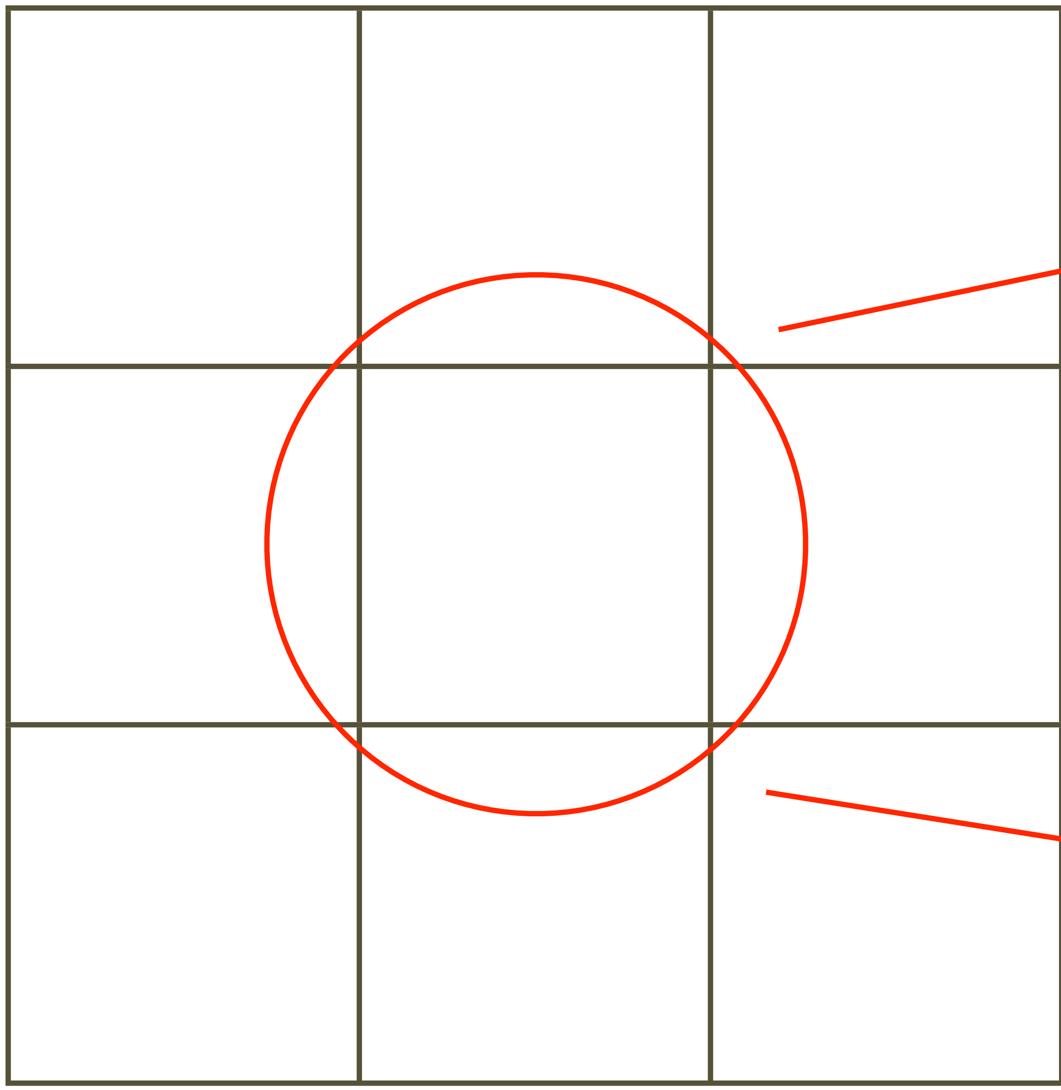
$$\mathcal{L}_{\rm GP} = f_\pi^2 m_\pi^2 + i\psi^*\partial_0\psi + \mu_{\rm eff}\,\psi^*\psi - \frac{g}{2}|\psi^*\psi|^2 + \psi^*\frac{\nabla^2}{2M}\psi$$

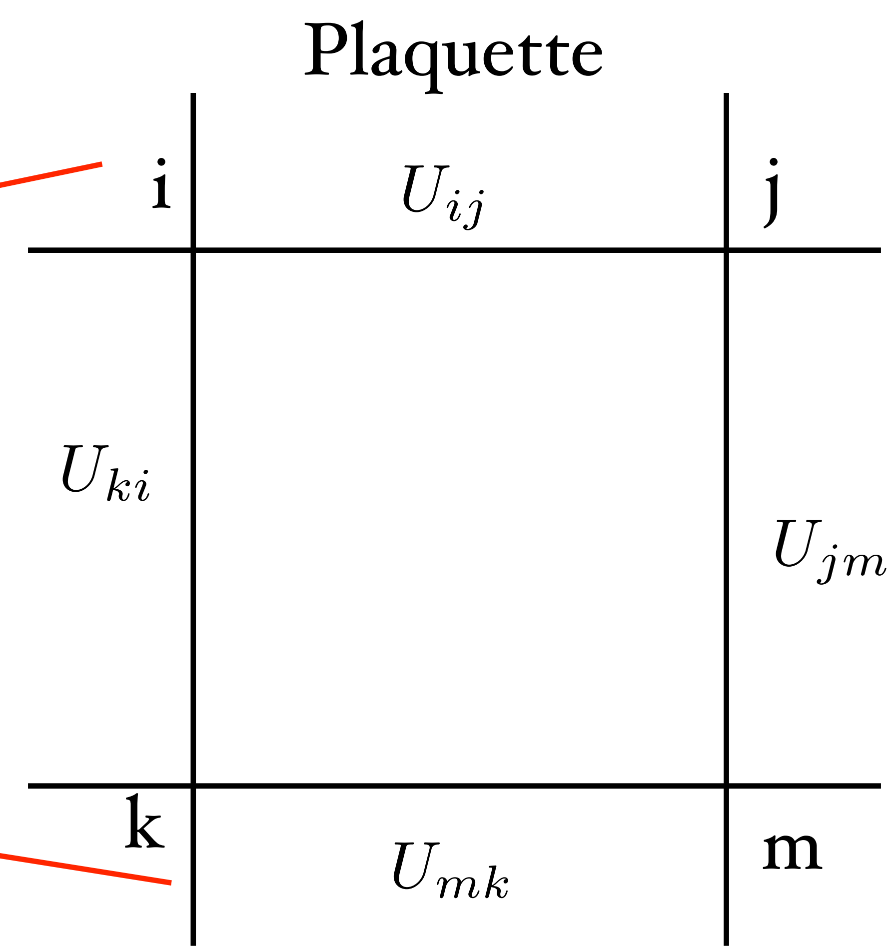
$$\mu_{\rm eff} = \frac{\mu_I^2 - m_\pi^2}{2\mu_I}\,, \qquad g = \frac{4\mu_I^2 - m_\pi^2}{12 f_\pi^2\mu_I^2}\,, \qquad M = \mu_I$$

S. Carignano, L. Lepori, G. Pagliaroli, A. Mammarella and M.M Eur.Phys.J. A53 (2017) no.2, 35

Lattice QCD

We cannot solve QCD in the continuum, but we can discretize and try to solve it





$U_{ij} \sim e^{iaA_\mu(x)}$

$$S(P) = \mathrm{Re}[\mathrm{Tr}(U_{ij}U_{jk}U_{km}U_{mi})]$$

$$S(P) \sim -\frac{1}{2}a^4\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

The generating functional

$$Z = \int dU e^{1/2g^2 \sum_P S(P)}$$

tends to the Yang-Mills in the continuum