Lattice study of rotating gluodynamics

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MISIS, JINR

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In collaboration with

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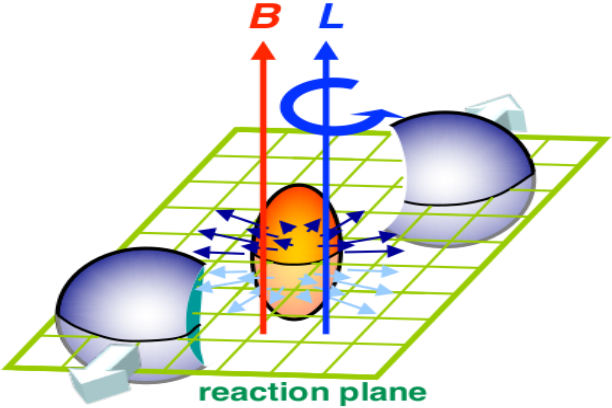
- D.D. Kuznedelev
- A.A. Roenko

V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. 112, 9–16 (2020)

Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

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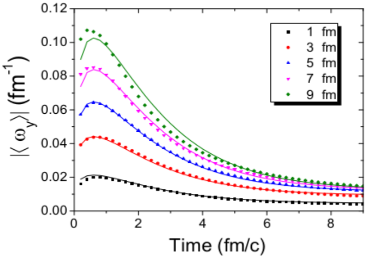

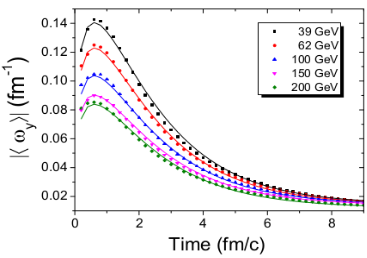

Hydrodynamic simulations (Phys.Rev.C 94, 044910 (2016))

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim (4-28)$ MeV ($\Omega \sim 20$ MeV $\Rightarrow v \sim c$ at distances 7 fm)
- Relativistic rotation of QGP

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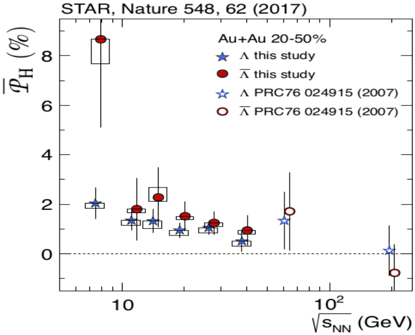

Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar{\Lambda}})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim (4 - 18)$ MeV
- Relativistic rotation of QGP

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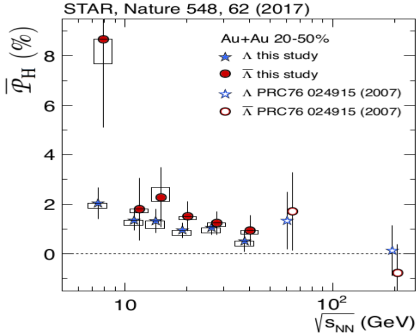

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How relativistic rotation influences QCD?

Recent works

- Arata Yamamoto, Yuji Hirono, Phys.Rev.Lett. 111 (2013) 081601
- S. Ebihara, K. Fukushima, K. Mameda, Phys. Lett. B 764 (2017) 94–99
- M.N. Chernodub, Shinya Gongyo, Phys.Rev.D 95 (2017) 9, 096006
- M.N. Chernodub, Shinya Gongyo, JHEP 01 (2017) 136
- Hui Zhang, Defu Hou, Jinfeng Liao, e-Print: 1812.11787 [hep-ph]
- Yin Jiang, Jinfeng Liao, Phys.Rev.Lett. 117 (2016) 19, 192302
- Xun Chen, Lin Zhang, Danning Li, Defu Hou, Mei Huang, e-Print: 2010.14478

...

Common features

- Mostly the studies are carried out in NJL (chiral transition)
- Critical temperature of the chiral phase transition drops with angular velocity
- Explanation: polarization of the chiral condensate (Phys.Rev.Lett. 117 (2016) 19, 192302)
- **Critical temperature of the confinement/deconfinement transition drops with angular velocity**

Study of rotating QGP

- Rotating QGP at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field

Study of rotating QGP

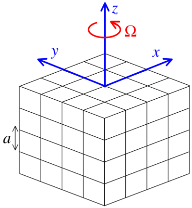
- Rotating QGP at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\, \exp\left[-\beta \hat{H}\right]$$

- Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2} \int d^4x \sqrt{g_E}\, g_E^{\mu\nu} g_E^{\alpha\beta} F_{\mu\alpha}^{(a)} F_{\nu\beta(a)}$$

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \mathrm{Tr}\big[(1 - r^2\Omega^2) F_{xy}^a F_{xy}^a + (1 - y^2\Omega^2) F_{xz}^a F_{xz}^a +$$

$$+(1 - x^2\Omega^2) F_{yz}^a F_{yz}^a + +F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz} F_{zy}\big]$$

Details of the simulations

- *Ehrenfest–Tolman effect:* **In gravitational field the temperature is not constant in space at thermal equilibrium**

$$T(r)\sqrt{g_{00}} = const = 1/\beta$$

$$T(r)\sqrt{1 - r^2\Omega^2} = 1/\beta$$

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- **One could expect that rotation decreases the critical temperature**
- We use the designation $T = T(r = 0) = 1/\beta$

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
  - Not appropriate for the field of velocities of rotating body
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$

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One can expect that boundary conditions influence our results, but their influence is restricted due to the screening

Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2}\int d^4x \mathrm{Tr}\big[(1-r^2\Omega^2)F^a_{xy}F^a_{xy} + (1-y^2\Omega^2)F^a_{xz}F^a_{xz} +$$

$$+(1-x^2\Omega^2)F^a_{yz}F^a_{yz} + +F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} -$$

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- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω ($\Omega < 50$ MeV)

Details of the simulations

The critical temperature

- Polyakov line

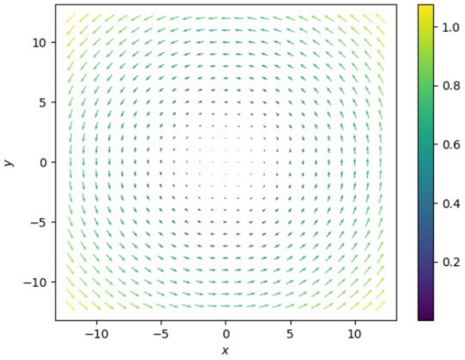
$$L = \left\langle \mathrm{Tr}\mathcal{T}\exp\left[ig\int_{[0,\beta]} A_4\, dx^4\right]\right\rangle$$

- Susceptibility of the Polyakov line

$$\chi = N_s^2 N_z \left(\langle |L|^2\rangle - \langle |L|\rangle^2\right)$$

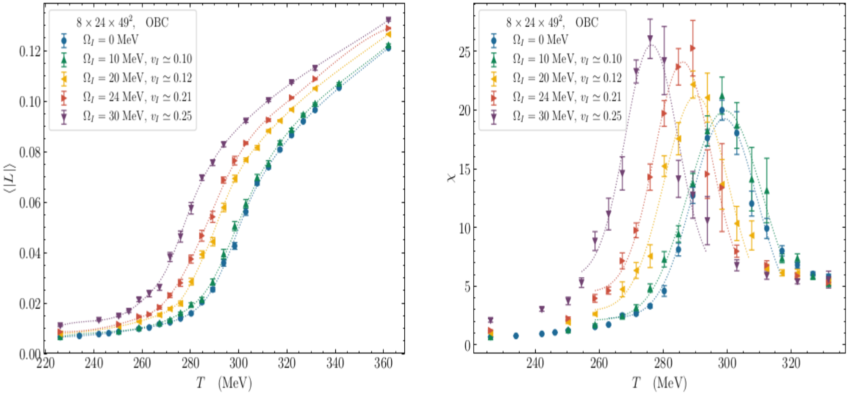
- T_c is determined from Gaussian fit of the $\chi(T)$

Rotation at zero temperature

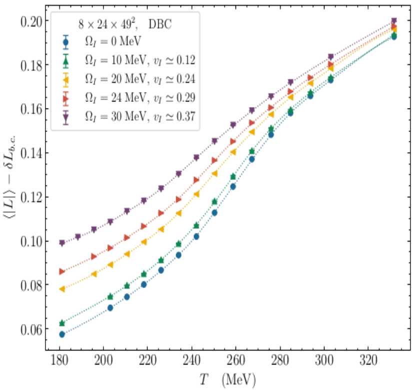


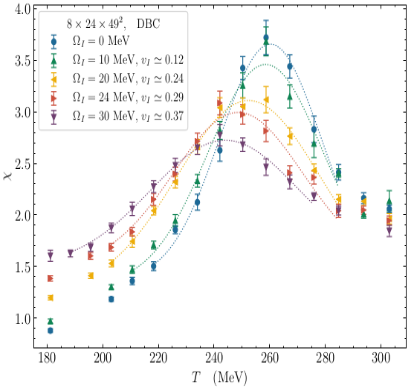
- $\langle trF_{\mu\nu}^2 \rangle \neq 0, \quad \langle T_{\mu\nu} \rangle = \epsilon g_{\mu\nu}, \quad \epsilon \sim \langle trF_{\mu\nu}^2 \rangle$
- In rotating frame $\langle T_{0i} \rangle \neq 0$
- The ground state of our system is "rotating vacuum"

Results of the calculation (Neumann b.c.)

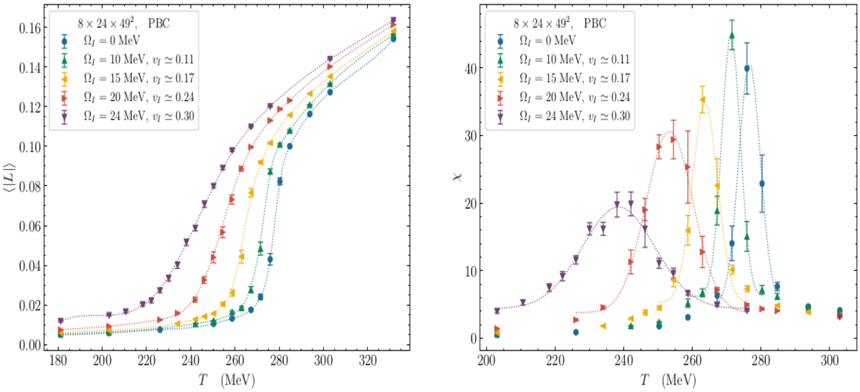


Results of the calculation (Dirichlet b.c.)

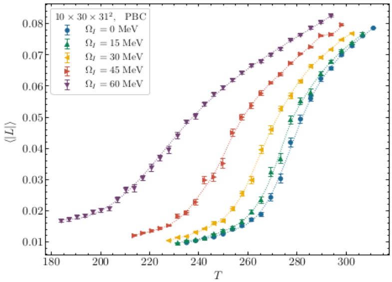


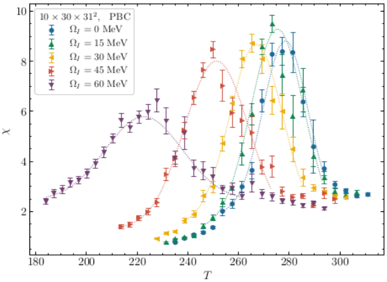


Results of the calculation (Periodic b.c.)



Results of the calculation





Volume dependence of the susceptibility

- **Periodic b.c.:** $\sim V$
- **Dirichlet b.c.:** $\sim const$
- **Neumann b.c.:** $\sim V$

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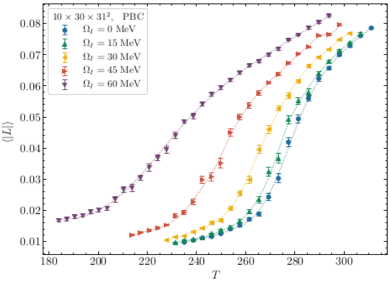

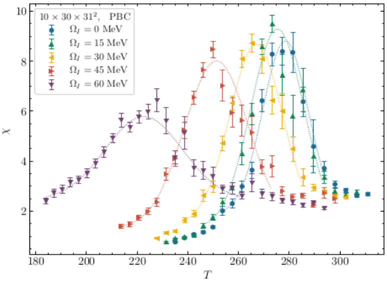

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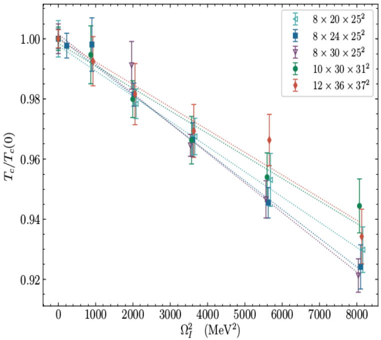
Rotation does not modify the order of the phase transition

Results of the calculation

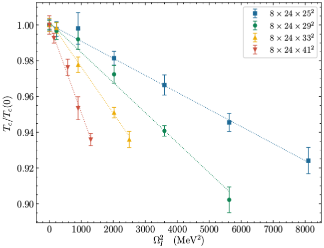


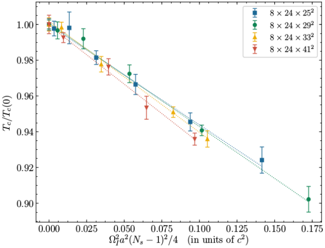
- The results can be well described by the formula ($C_2 > 0$)

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

- **The critical temperature rises with angular velocity**
- The results weakly depend on lattice spacing and the volume in z-direction

Dependence on the transverse size





- The results can be well described by the formula

$$\frac{T_c(\Omega)}{T_c(0)} = 1 - B_2 v_I^2, \quad v_I = \Omega_I (N_s - 1) a/2, \quad C_2 = B_2 (N_s - 1)^2 a^2/4$$

- **Periodic b.c.:** $B_2 \sim 1.3$
- **Dirichlet b.c.:** $B_2 \sim 0.5$
- **Neumann b.c.:** $B_2 \sim 0.7$

Conclusion

- We have carried out lattice study of how relativistic rotation influences confinement/deconfinement transition
- Critical temperature of the confinement/deconfinement transition rises with Ω
- Critical temperature of the chiral transition drops with Ω
- One needs to include dynamical quarks to see who wins

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THANK YOU!